Who Is Mad?

“In that direction,” the Cat said, waving its right paw round, “lives a Hatter; and in that direction,” waving the other paw, “lives a March Hare. Visit either you like: they’re both mad.”

“But I don’t want to go among mad people,” Alice remarked.

“Oh, you can’t help that,” said the Cat: “we’re all mad here.”

—Alice’s Adventures in Wonderland, CHAPTER SIX

Shortly after the trial, Alice met the Duchess, and they had the following remarkable conversation.

“The Cheshire Cat says that everyone here is mad,” said Alice. “Is that really true?”

“Of course not,” replied the Duchess. “If that were really true, then the Cat would also be mad, hence you could not rely on what it said.”

This sounded perfectly logical to Alice.

“But I’ll tell you a great secret, my dear,” continued the Duchess. “Half the creatures around here are mad—totally mad!”

“That doesn’t surprise me,” said Alice, “many have seemed quite mad to me!”

“When I said totally mad,” continued the Duchess, quite ignoring Alice’s remark, “I meant exactly what I said: They are completely deluded. All their beliefs are wrong—not just some, but all. Everything true they believe to be false and everything false they believe to be true.”

Alice thought awhile about this very queer state of affairs. “Does a mad person or creature believe that two plus two equals five?” asked Alice.

“Why, of course, child! Since two plus two doesn’t equal five, then a mad person naturally believes that it does.”

“And does a mad person also believe that two plus two equals six?”

“Of course,” replied the Duchess, “since it doesn’t, then the mad one believes it does.”

“But it can’t both equal five and six!” exclaimed Alice.

“Of course not,” agreed the Duchess, “you know that and I know that, but a mad person doesn’t. And the moral of that is—”

“What about the sane people around here?” interrupted Alice (who had heard quite enough morals for the day). “I guess most of their beliefs are right but some of them are wrong?”

“Oh, no, no!” said the Duchess most emphatically. “That may be true where you come from, but around here the sane people are one hundred percent accurate in their beliefs! Everything true they know to be true and everything false they know to be false.”

Alice thought this over. “Which ones around here are sane and which ones are mad?” asked Alice. “I would very much like to know this.”

THE CATERPILLAR AND THE LIZARD “Well,” replied the Duchess, “take, for example, the Caterpillar and Bill the Lizard. The Caterpillar believes that both of them are mad.”

“Which of them is really mad?” asked Alice.

“I shouldn’t have to tell you that!” replied the Duchess. “I have given you enough information for you to deduce the answer.”

What is the solution? Is the Caterpillar mad or sane? And what about the Lizard?

THE COOK AND THE CAT “Then there’s my cook and the Cheshire Cat,” continued the Duchess. “The Cook believes that at least one of the two is mad.”

What can you deduce about the Cook and the Cat?

THE FISH-FOOTMAN AND THE FROG-FOOTMAN “That was very interesting,” said Alice. “The two cases are quite different.”

“Of course they are, my dear! And the moral of that is—to be or not to be is not the same as to be and not to be.”

Alice tried to figure out just what the Duchess had meant, when the Duchess interrupted her thoughts.

“Then there are my two footmen, the Fish-Footman and the Frog-Footman. You’ve met them?”

“Oh, yes, indeed!” said Alice, remembering the latter’s unspeakable rudeness.

“Well, the Fish-Footman believes that he and the Frog-Footman are alike—in other words that they are either both sane or both mad. And now, my dear, it is up to you to tell me which ones are mad.”
Alice did not quite see why it should be up to her. Still, the puzzle interested her, so she worked on it quite a while.

"I'm afraid I can't solve it," said Alice, "I know what one of the footmen is, but I cannot figure out the other."

"Why, you have solved it, you dear thing!" said the Duchess, giving Alice a hug. "The other footman can't be figured out from what I have told you. In fact, even I don't know what the other one is."

Which footman do you know to be sane or mad, and what is he?

THE KING AND QUEEN OF DIAMONDS "Then there's the King and Queen of Diamonds," began the Duchess.

"The King and Queen of Diamonds?" said Alice. "I don't believe I've met them—in fact I did not know they were around here."

"All the cards are around here," said the Duchess. "Anyway, I heard a rumor that the Queen of Diamonds was mad. However, I was not sure whether the person who told me this was mad or sane, so I decided to find out for myself.

"Well, one day I met the King of Diamonds without his Queen. I knew him to be absolutely honest, though of doubtful sanity, hence whatever he would say he would at least believe to be true.

"Is your poor dear wife really mad?" I asked sympathetically."

"She believes she is," replied the King."

What can be deduced about the King and Queen of Diamonds?

WHAT ABOUT THESE THREE? "I've always wondered about the March Hare, the Hatter, and the Dormouse," said Alice. "The Hatter is called the Mad Hatter, but is he really mad? And what about the March Hare and the Dormouse?"

"Well," replied the Duchess, "the Hatter once expressed the belief that the March Hare does not believe that all three of them are sane. Also, the Dormouse believes that the March Hare is sane."

What can you deduce about these three?

AND THESE THREE? "Then there's the Gryphon, the Mock Turtle, and the Lobster," the Duchess began.

"I didn't know there was a real lobster around here," replied Alice. "I only know him in a poem."

"Oh, yes, there is a real lobster, and he is as big as the Mock Turtle," replied the Duchess. "Anyway, the Lobster once expressed the belief that the Gryphon believes that exactly one of the three is sane. The Mock Turtle believes that the Gryphon is sane."

What can you deduce about these three?

AND NOW, WHAT ABOUT THESE TWO? "You know," said Alice in a very low voice, looking around to see that the Queen of Hearts was not within hearing distance, "I am particularly interested in knowing about the King and Queen of Hearts. What are they?"

"Ah," said the Duchess, "this is an interesting story indeed! The Queen believes that the King believes that the Queen believes that the King believes that the Queen is mad."

"Now that's too much!" cried Alice. "I think I'll go mad if I try to puzzle that one out!"

"Very well," said the Duchess good-naturedly, "let's try an easier one first. For example, take the King and Queen of Spades."

There was a long pause.

"What about the King and Queen of Spades?" asked Alice.

"Well, the Queen believes that the King believes that she is mad. What can you tell me about the King and Queen of Spades?"

THE KING AND QUEEN OF CLUBS "You got that one pretty easily," said the Duchess. "Now, what would you say if I told you that the King of Clubs believes that the Queen of Clubs believes that the King of Clubs believes that the Queen of Clubs is mad?"
AND NOW, WHAT ABOUT THE QUEEN OF HEARTS? Alice thought this last puzzle over and said, "If you had told me that (which of course you didn't) then I'm afraid I would have had to conclude that you must be mad!"

"And quite right you would be!" cried the Duchess. "But of course I would never tell you any such impossible thing!"

"And now," continued the Duchess, "you should be able to solve the puzzle about the King and Queen of Hearts. Remember what I told you: The Queen believes that the King believes that the Queen believes that the King believes that she is mad. The question is, Is the Queen of Hearts mad or sane?"

"Well," said the Duchess, "Three believes that One is mad. Four believes that Three and Two are not both mad. Five believes that One and Four are either both mad or both sane. Six believes that One and Two are both sane. Seven believes that Five is mad. As for the Knave of Hearts, he believes that Six and Seven are not both mad.

"And now," continued the Duchess, "would you care to figure out whether the Knave is mad or sane, or would you prefer a more confusing puzzle?"

"Oh, no," replied poor Alice, "this one is quite confusing enough, thank you!"

Is the Knave of Hearts mad or sane?

THE DODO, THE LORY, AND THE EAGLET "Then there's the Dodo, the Lory, and the Eaglet," said the Duchess. "The Dodo believes that the Lory believes that the Eaglet is mad. The Lory believes that the Dodo is mad, and the Eaglet believes that the Dodo is sane.

"Can you puzzle that out?" asked the Duchess.

THE KNAVE OF HEARTS Alice solved this last puzzle.

"I think I know why half the people around here are mad," said Alice.

"Why?" asked the Duchess.

"I think they went mad trying to work out puzzles like these. They're dreadfully confusing!"

"As to confusing puzzles," replied the Duchess, "these are nothing compared to some I could tell you if I chose!"

"Oh, you needn't choose!" said Alice as politely as she could.

"For example, there's the Knave of Hearts," the Duchess went on, "he keeps company with the Spade-Gardeners, One, Two, Three, Four, Five, Six, and Seven. I believe you've met Two, Five, and Seven?"

"Oh, yes," remembered Alice, "they were having a terrible time trying to paint the white roses red, because they had by mistake planted a white rose tree in the garden instead of a red rose tree as the Queen had ordered."

THE GRYPHON'S EVALUATION "You know," said the Duchess, chuckling, "it's a funny thing—really a funny thing!"

"What is?" asked Alice.

"Why, the Cook—she thinks that I am mad. Isn't that a scream?"

Alice did not see why this was so funny.

"Anyway, my dear," continued the Duchess, "I must now be off to the croquet game. It was delightful talking to you again."

After the Duchess left, Alice stood quite a while in thought. She was thinking so deeply that she did not even notice the Gryphon, who had just come by.

"What you puzzling so hard?" asked the Gryphon.

Alice then told the Gryphon her entire conversation with the Duchess.

"It's all her fancy, it is," said the Gryphon, chuckling. "You can't believe her story, you can't."

"Why not?" asked Alice.

"Because it don't hang together, nohow—it just don't make no sense! It's just her fancy, I tells you!"

Alice thought this over. "Is it possible the Duchess was lying?" asked Alice.

"No, she wasn't lying," he replied, "she just fancied the whole story—she fancies lots of things!"

The Gryphon then explained to Alice why the Duchess's story as a whole is impossible, and he was right! If you review all the things the Duchess said, you will see that it is not consistent (assuming the Duchess was not deliberately lying).

How does one prove this?