MATH 433 – Quiz 1 (Due Jan 27)

1. a) (3 pts) Prove that if A, B are sets, then
   \[ P(A \cap B) = P(A) \cap P(B) \]
   b) (3 pts) Is it always true that \( P(A \cup B) = P(A) \cup P(B) \)?
   Justify your answer.

2. a) (2 pts) Compute \( A \Delta \emptyset, A \Delta \emptyset, A \Delta A, A \Delta A \emptyset \).
   Justify your answers.
   b) (3 pts) On a Venn Diagram shade \( (A \Delta B) \Delta C \).
   Is the symmetric difference \( \Delta 
   \) associative?
   Justify your answer.
   c) (2 pts) Show that the symmetric difference \( \Delta \)
   satisfies the cancellation law, namely if \( A \Delta B = A \Delta C \) then \( B = C \).

3. (6 pts) Use the Inclusion–Exclusion Principle to compute the number of integers in the interval
   \([1, 10000]\) which are neither divisible by 2 nor by 3 nor by 5.

4. Let \( f : \mathbb{R}^2 \to \mathbb{R}^2 \) be defined by:
   \[ f(x, y) = (2x - 3y, 3x + 2y) \]  
   a) (2 pts) Show that \( f \) is a bijection.
   b) (2 pts) Find a formula for \( f^{-1}(x, y) \).
   c) (2 pts) Let \( f : \mathbb{Z}^2 \to \mathbb{Z}^2 \) be defined by \((x, y)\).
   Is \( f \) an injection? a surjection? Justify your answers.