

## Freshman-Sophomore Contest 2004

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4. Find the first four nonzero terms in the MacLaurin series expansion  $F(x) = \sum a_k x^k$  of the function  $F(x)$  defined by

$$F(x) = \int_0^x e^{-t^2} dt.$$

5. Let  $g(x) = \max[1 - |x|, 0]$ .

(a) Graph  $g(x)$  on  $-2 \leq x \leq 2$ .

(b) Let

$$h(x) = \int_{u=-1/2}^{1/2} g(x-u) du.$$

Evaluate  $h(1/4)$ .

(c) Sketch the graph of  $h(x)$  on  $-2 \leq x \leq 2$ .

6. Consider the function  $f(x) = (1 - x^2)^{10} x^{20}$ .

(a) Find the value of  $x$  in  $[0, 1]$  that maximizes  $f(x)$ .

(b) Prove that  $\ln[(1 - x^2)^{10} x^{20}]$  is concave down on the open interval  $(0, 1)$ .