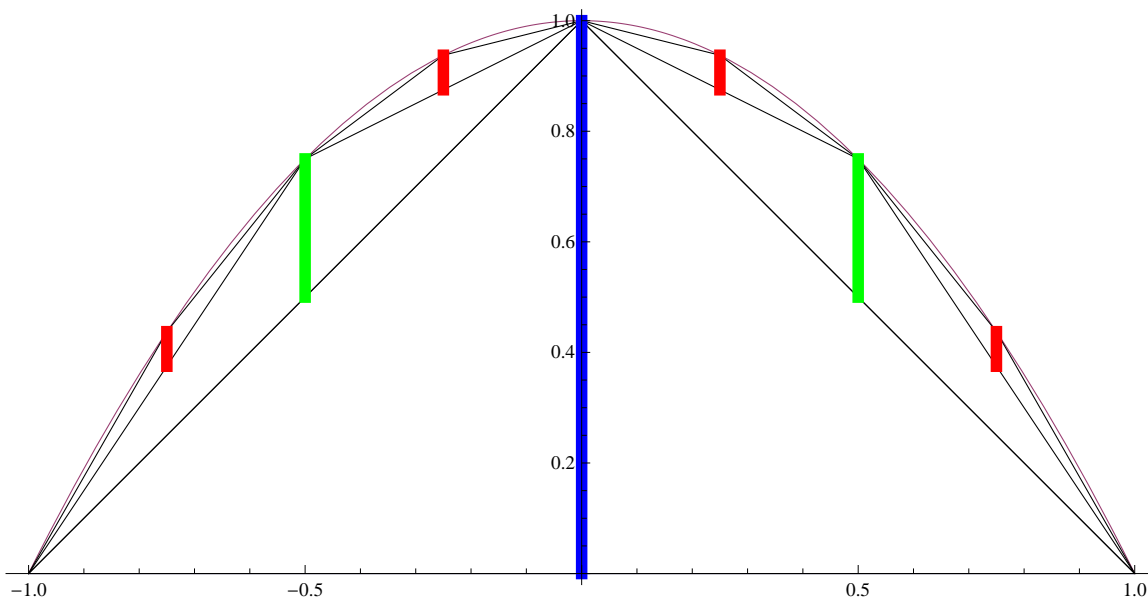


# Geometric series through time

Archimedes and the parabola.

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## Archimedes and the area enclosed by a parabola and chord.



Archimedes got the area, here  $\frac{4}{3}$ , by the following analysis. The first triangle has a certain area, readily calculated. The next two triangles, the green ones, each have height  $\frac{1}{4}$  the blue one, and base half the blue one, so each green-height triangle has  $\frac{1}{8}$  the area of the blue-height triangle. But then, there are two of them. The greens give  $\frac{1}{4}$  the area of the blue. In like manner, the reds give area  $\frac{1}{4}$  the greens, and so on. Geometric series, posit a value  $X$  for the total, and observe that  $X=1+X/4$ . Thus  $X=\frac{4}{3}$ .

Of course, there's a lot of geometry under the hood here. Nowadays, we use a lot of algebra in our geometry, and things would go like this:

The height of a triangle nestled into a chord going from  $(a, 1-a^2)$  to  $(b, 1-b^2)$ , with third vertex at  $(\frac{a+b}{2}, 1-(\frac{a+b}{2})^2)$ , is the vertical distance between  $(1-(a+b)^2/4)$  and  $(1-(a^2+b^2)/2)$ , and algebraically, that works out to  $(a^2+b^2)/2 - (a+b)^2/4 = (b-a)^2/4$ .

So the height is proportional to the square of the horizontal span of the chord. Thus, halving the chord quarters the height and multiplies the area by  $\frac{1}{8}$ .

**Under the hood: (click on menu to open closed cells).**

```
pix1 = Plot[{0, 1 - x^2}, {x, -1, 1}, AspectRatio -> Automatic]
```

