Math 172 Exam I Summary Sheet and Sample Stuff

(NOTE: The questions posed here are not necessarily a guarantee of the type of questions which will be on Exam I. This is a sampling of questions I have asked on previous exams over this material. These are simply provided as an additional source of practice.)

DEFN indicates a definition, theorem, or concept to remember.
FORM indicates a formula to remember.

Section 6.2-6.3
1. Riemann Sum Definition of the Definite Integral (DEFN)
2. Equally Spaced Definition of the Definite Integral (DEFN: GIVEN if needed)
3. Relationship of Definite Integral to Area \((f > 0)\) (DEFN)
4. Approximating Riemann Sums
5. Writing Riemann Sums as Integrals
6. Potential Proof Topics:
   a) Properties of Definite Integrals (#1-4, 6-9 in 6.3)
   b) General Integrals Using Riemann Sums (like 6.3 #21-22: Summation Formulas given)
6. Sample Problems:
   a) Approximate \(\int_1^3 \frac{1}{x} \, dx\) using \(P = \{1, 1.25, 1.5, 2, 3\}\) and taking \(x_i^*\) to be the left endpoint of each subinterval.
   b) Compute \(\lim_{n \to \infty} \frac{2}{n} \sum_{i=1}^{n} \left[ \left( 1 + \frac{2i}{n} \right)^2 - \left( 1 + \frac{2i}{n} \right) \right]\)
   c) Use the definition of the definite integral to find the exact value of \(\int_0^1 (1 - x^2) \, dx\).

Section 6.4-6.5
1. Fundamental Theorem of Calculus, pt 1 (DEFN)
2. Fundamental Theorem of Calculus, pt 2 (DEFN)
3. Compute Integrals using FTC pt 2
4. Substitution
5. Boundaries of a Definite Integral using Substitution
6. Potential Proof Topic: FTC part 1 and 2
7. Sample Problems:

   Compute the following integrals:
   a) \(\int_0^2 x\sqrt{4x^2 + 9} \, dx\)
   b) \(\int_0^2 x^2\sqrt{x^3 + 1} \, dx\)
   c) \(\int_\frac{\pi}{4}^\frac{\pi}{2} \tan x \, dx\)
Section 6.6
1. Integral Definition of Logarithm (GIVEN if needed)
2. Definition of Exponential as Inverse of Logarithm
3. Potential Proof Topics:
   a) Properties of Logarithms Using Integral Definition
   b) Laws of Exponents (see p412 for example)

Section 7.1
1. Area Between Curves
2. Area integrating w/r/t y
3. Potential Proof Topic: Deriving Formulas for Area Using Riemann Sums
4. Sample Problems:

   a) Find the area of the region bounded by the curves \(y = x - 1\) and \(y^2 = 3 - x\)

   b) Find the area of the region bounded by the graphs of \(y = \frac{1}{x}\), \(y = \frac{1}{x^2}\), and \(x = 2\).

   c) Find the area of the region bounded by the graphs of \(y = \sqrt{2 - x}\), \(y = -\sqrt{2 - x}\), and \(y = -x\).

Section 7.2-7.3
1. Volume using Slices (FORM: \(V = \pi r^2 h\))
2. Volume using Cylindrical Shells (FORM: \(V = 2\pi rh\Delta r\))
3. Volume rotating a region about a different axis (\(x = k\) or \(y = k\))
4. Writing Riemann Sums to represent volumes of solids
5. Sample Problems:

   a) Find the volume of the solid formed by rotating the region in the first quadrant bounded by \(y = x\) and \(y = x^3\) about the \(x\)-axis; about \(x = -1\)

   b) Find the volume of the solid obtained by rotating the region bounded by \(y = \sqrt{x}\), \(x = 0\), and \(y = 3\) about the \(x\)-axis.

   c) The portion of the circle \(x^2 + y^2 = 4\), \(x \in [1, 2]\) is rotated about the \(y\)-axis to produce a napkin ring. Find the volume of the ring.

Section 7.4
1. Work: force a function of displacement (Hooke’s Law (FORM), etc)
2. Potential Proof Topic: Derive work formula when force is a function of displacement
3. Sample Problems:

   a) A spring has a natural length of 12 inches (1 ft). The force required to keep the spring stretched to a length of 16 inches is 30 lbs. Find, in ft-lbs, the work required to stretch the spring from a length of 12 inches to a length of 18 inches.

   b) A cable weighing 2 lb/ft is used to lift 100 lbs of coal up a mine shaft 100 feet deep. Find the total work done, in ft-lbs, in moving the coal 90 feet.
Section 7.5
1. Average Value of a Function (DEFN)
2. Geometric Interpretation of $f_{avg}$ (DEFN)
3. Mean Value Theorem for Integrals (DEFN)
4. Potential Proof Topics:
   a) Derivation of Formula for $f_{avg}$
   b) MVT for Integrals
5. Sample Problems:
   a) Find the value of $b$ such that the average value of $f(x) = 4 - x^2$ from $x = 0$ to $x = b$ is equal to 1
   b) On a certain day, the temperature $t$ hours past 10am is given by $T(t) = 80 + 10 \sin \left( \frac{\pi t}{12} \right)$. What was the average temperature between 2pm and 6pm?
   c) Find the average value of $f(x) = \sin x$ on the interval $[0, \pi]$. 