Solutions to HW 3

#1 Solve the x equation for t and substitute into the y equation:

\[ x = 25\sqrt{3}t, \quad t = \frac{x}{25\sqrt{3}} \]

\[ y = -4.9 \left( \frac{x}{25\sqrt{3}} \right)^2 + \frac{x}{25\sqrt{3}} + 20 \]

\[ y = -\frac{4.9x^2}{125\sqrt{3}} + \frac{x}{25\sqrt{3}} + 20 \]

To determine where the cannonball hits, we need to find x when y = 0. Using the quadratic formula, we obtain:

\[ 0 = -\frac{4.9x^2}{125\sqrt{3}} + \frac{x}{25\sqrt{3}} + 20 \]

\[ x = \frac{-\frac{1}{\sqrt{3}} + \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 - 4 \left(\frac{-4.9}{125\sqrt{3}}\right) 20}}{2 \left(-\frac{4.9}{125\sqrt{3}}\right)}, \quad x = \frac{-\frac{1}{\sqrt{3}} + \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 - 4 \left(\frac{-4.9}{125\sqrt{3}}\right) 20}}{2 \left(-\frac{4.9}{125\sqrt{3}}\right)} \]

\[ x = 251.4 \text{ meters (other solution invalid since it is negative)} \]

#2 First, we have to simplify the expression since the vertical asymptotes ONLY occur where the denominator approaches 0 but the numerator does not:

\[ \frac{x^2 + 5x - 24}{x^2 - 9} = \frac{x^3}{(x-3)(x+8)} \]

So the only vertical asymptote occurs when x = -8. The one-sided limits are:

\[ \lim_{x \to -8^-} \frac{1}{x+8} = -\infty \]

\[ \lim_{x \to -8^+} \frac{1}{x+8} = \infty \]

#3 (Proof shown here; scratchwork done in class). Let f(x) = x^2 + 3x, a = 1, and L = 4. Pick 0 < \varepsilon and let \delta = \min(1, \frac{\varepsilon}{6})

If |x - 1| < \delta, \quad |x - 1| < \frac{\varepsilon}{6}, \quad 6|x - 1| < \varepsilon

Since |x - 1| < \delta, \quad |x - 1| < 1

as well, so we must have -1 < x - 1 < 1, or

0 < x < 2

4 < x + 4 < 6

Therefore, we have -x + 4 < 6. This means

|x + 4|, \quad |x - 1| < 6 \quad |x - 1| < \varepsilon \quad \text{and} \quad 6|x - 1| < \varepsilon

so we must have |(x + 4)(x - 1)| < \varepsilon

\[ |x^2 + 3x - 4| < \varepsilon \]

\[ |f(x) - L| < \varepsilon \]

Therefore, by definition, \( \lim_{x \to 1} x^2 + 3x = 4 \)

Answers to Even-numbered text problems:

1.3 #4 \( y = (\frac{x-1}{2})^2 - 1 \) or \( y = \frac{4x^2}{4} + \frac{16x}{4} - \frac{1}{4} \)

1.3 #18 start and end (0, 3); travel the ellipse \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \) clockwise

1.3 #26 \( r(t) = <3+5t, 4+4t> \) (answers may vary)

2.2 #4a) 0 b) DNE c) 1 d) 0 e) 1 f) DNE

2.4 #12 take \( \delta = \frac{\varepsilon}{2} \)

#22 take \( \delta = e^{\frac{\varepsilon}{2}} \)
#36 take $\delta = \frac{1}{M^*}$