Texas Geometry and Topology Conference

This is a report on the presentations at the 51st meeting of the Texas Geometry and Topology Conference at Texas Tech University on February 14-16, 2014. This conference was partially supported by National Science Foundation Grant DMS-1203131, and Texas Tech University. Speakers reported on recent research. All plenary speakers provided abstracts. Plenary speakers were encouraged to offer in their abstracts slightly broader discussions of the significance and context of their results.

Meeting 51. Texas Tech University, February 14-16, 2014

Richard Hind, University of Notre Dame, *Symplectic embeddings and isotopies*

A goal of symplectic topology is to determine in which situations symplectomorphisms have properties similar to volume preserving maps, that is, they are very flexible, and in which situations they exhibit rigidity as in Kähler geometry. Flexibility results have been established using versions of the $h$-principle, while rigidity in symplectic topology is a consequence of the theory of pseudoholomorphic curves. Basic testing areas for borderlines between rigidity and flexibility are embedding and isotopy problems for domains in the standard symplectic Euclidean space. Given two domains $U$ and $V$, we ask if there exists a symplectic embedding (or a Hamiltonian flow) mapping $U$ into $V$. Given two embeddings, we can ask whether they are isotopic through symplectic embeddings. First, we will review some results on the embedding problem focusing on the cases when $U$ is an ellipsoid or polydisk and $V$ is a ball. Then we discuss some recent results on the isotopy problem. For both problems a construction known as symplectic folding gives sharp estimates.

Thang Le, Georgia Institute of Technology, *Symplectic embeddings and isotopies*

The Habiro ring first appeared in work of Habiro where he showed that there is a unified Witten-Reshetikhin-Turaev (WRT) $sl_2$ invariant of integral homology 3-spheres with values in the Habiro ring. The Habiro ring has attracted attention of number theorists (Manin, Marcolli, . . . ) who suggest that the Habiro ring can be considered as the ring of regular functions on the elusive ”field with 1 element”. In the talk we will explain the Habiro ring and show that for any simple Lie agebra, there is a unified quantum invariant of integral homology 3-spheres with values in the Habiro ring which reduces to the WRT invariant at roots of unity. This is joint work with Habiro.

Liviu Nicolaescu, University of Notre Dame, *Random Morse functions on compact manifolds*

It is well known that any smooth function on a smooth compact Riemann manifold admits a Fourier-like decomposition as a superposition of eigenfunctions of the Laplacian. If the Fourier coefficients are random, and their randomness is chosen appropriately, then the resulting random function is almost surely a Morse function. I will describe the statistics of the sets of critical points and critical values of such a random Morse function. Additionally I will describe how to reconstruct the Riemannian geometry of the manifold from statistics of such a random function. The method of investigation is based on a blend of geometric and probabilistic ideas which reduce the investigation to the spectral estimates of Hrmander and some classical results concerning random symmetric matrices.

Peter Olver, University of Minnesota, *Moving frames and their applications*

The classical method of moving frames was developed by Elie Cartan into a powerful tool for studying the geometry of submanifolds under certain geometrical transformation groups. In this talk, I will present a
new foundation for moving frame theory based on equivariant maps. The method is completely algorithmic, and applies to very general Lie group actions and even infinite-dimensional pseudo-groups. It has led to a wide variety of new applications, ranging over classical differential geometry, differential equations, the calculus of variations, geometric flows, image processing, invariant numerical algorithms, invariant theory, and elsewhere. The talk will survey the key ideas, and present some of the principal applications.

**Peter Olver, University of Minnesota, Object recognition, symmetry detection, jigsaw puzzles and melanomas**

*(General Audience Talk)* I will survey developments in the application of invariants of various types, including invariant histograms and differential invariant signatures, for object recognition and symmetry detection in images. I will present some recent progress, including applications to automated jigsaw puzzle assembly and melanoma detection.

**Richard Schoen, Stanford University, Trapped surfaces and MOTS in general relativity**

The study of the Einstein equations leads to some interesting geometric problems. In this talk we will describe one of these, the theory of marginally outer trapped surfaces (MOTS), which may be viewed as a generalization of minimal surface theory to a Lorentz setting. These surfaces play an important role in the theory of the Einstein equations, first in singularity formation, and then in the study of gravitational energy and momentum. We will explain how these surfaces arise, the way they can be constructed, and the notion of stability which parallels in a remarkable way the stability condition for minimal hypersurfaces. We will also describe some of the recent applications to relativity.

**Susan Tolman, University of Illinois at Urbana-Champaign, The cohomology of quotients of Hamiltonian loop group spaces**

I will describe a new way to compute the Poincare series of the moduli space of flat bundles over a Riemann surface with prescribed holonomies at punctures. This is an example of a more general theory which computes the Poincare series of the quotient of a (proper) Hamiltonian loop group action. The proof is analogous to the proof in the compact case, and relies on Morse theory. Based on work with Bott and Weitsman.

**Zhenghan Wang, Microsoft, Quantum topology and its applications**

Quantum topology and quantum algebra have found applications in topological phases of matter and quantum computation. This topological approach is pursued in Microsoft Station Q, where topological quantum field theories (TQFTs) and modular categories play an essential role. I will focus on some new lattice models for (3+1)-TQFTs by K. Walker and myself, which have been used recently to generalize topological insulators.

**Igor Zelenko, Texas A&M University, Symplectic methods in local differential geometry of nonholonomic distributions**

I would like to describe a new method for the construction of local differential invariants and canonical frames for a wide class of bracket-generating vector distributions (subbundles of tangent bundles) under the natural action of the group of diffeomorphisms of the ambient manifolds. The method consists of passing to the dual objects, which are certain odd dimensional submanifolds of the cotangent bundle. Making the quotient of the dual submanifold by its characteristic rank 1 foliation (locally near generic points), we assign to each leaf of this foliation a curve of flags of isotropic and coisotropic subspaces in a linear symplectic
space. Studying the differential geometry of such curves of flags with respect to the action of the linear symplectic group we were able to construct differential invariants and canonical frames for distributions of arbitrary rank satisfying very mild genericity assumptions. These constructions are much more unified and are applied to much larger classes of distributions than all previous constructions based on the classical Cartan method of equivalence. In the talk we will concentrate mostly on the case of rank two distributions that is reduced by this approach to the classical differential geometry of curves in projective spaces. The synopsis of the general case of arbitrary rank will be given as well.