Texas Geometry and Topology Conference

This is a report on the presentations at the 48th meeting of the Texas Geometry and Topology Conference at Rice University on November 9-11, 2012. This conference was partially supported by National Science Foundation Grants DMS-0904481 and DMS-1203131, and Rice University. Speakers reported on recent research. Both plenary speakers and participants in the associated poster session provided abstracts. Plenary speakers were encouraged to offer in their abstracts slightly broader discussions of the significance and context of their results.

Meeting 48. Rice University, November 9–11, 2012

Matthew Bainbridge, Indiana University, *Effective Veech dichotomy and diagonal flows*

The Veech dichotomy tells us that for certain very symmetric flat surfaces, we have a nearly perfect understanding of the dynamics of the geodesic flow: in every direction, the geodesic flow is either uniquely ergodic or periodic. In this talk, we discuss a more effective version of the Veech dichotomy which allows one to control the geometry of periodic orbits in any direction. The proof involves geometry of numbers, and dynamics of diagonal flows on homogeneous spaces over the adeles. This is joint work with Martin Möller.

Tom Church, Stanford University, *Stability in the unstable cohomology of mapping class groups, SL_n(Z), and Aut(F_n)*

For each of the sequences of groups in the title, the kth rational cohomology is known to be independent of n in a linear range \( n \geq c \cdot k \). Furthermore, this “stable cohomology” has been explicitly computed in each case. In contrast, very little is known about the unstable cohomology, which lies outside this range.

In this talk I will explain a conjecture on a new kind of stability in the cohomology of these groups, joint with Benson Farb and Andrew Putman. These conjectures concern the unstable cohomology, in a range near the “top dimension” (the virtual cohomological dimension), and for \( SL_n(Z) \) they imply that the unstable cohomology vanishes in that range. One key ingredient is a version of Poincare duality for these groups based on the topology of the curve complex and the algebra of modular symbols. Very recently, Avner Ash has announced a proof of our conjecture for \( SL_n(Z) \) (unpublished).

Gavril Farkas, Humboldt University, *Classification of universal Jacobians over the moduli space of curves*

The universal Jacobian \( J_g \) is the fibration over the moduli space of curves with fibres being Jacobian varieties of curves of genus \( g \). I will discuss joint work with Verra in which a complete classification of \( J_g \) by Kodaira dimension has been carried out. Thus \( J_g \) is a unirational variety when \( g < 9 \), has Kodaira dimension zero (respectively 19) when \( g = 10 \) (respectively \( g = 11 \)), and is of Kodaira dimension \( 3g - 3 \) for all other cases.

Samuel Grushevsky, Stony Brook University, *Stable cohomology of the perfect cone toroidal compactification of \( A_g \)*

The cohomology of \( A_g \), the moduli space of principally polarized complex \( g \)-dimensional abelian varieties, is the same as the cohomology of the symplectic group with integer coefficients \( Sp(2g, Z) \). The stable cohomology \( H^k(A_g) \) for \( g > k \) was computed by Borel; the stable cohomology of the Satake-Baily-Borel compactification of \( A_g \) was computed by Charney and Lee using topological methods. In a joint work with
Klaus Hulek and Orsola Tommasi we show that the cohomology of the perfect cone toroidal compactification of $A_g$ stabilizes, and compute some of this stable cohomology using algebro-geometric methods. There is also an independent related work in progress, by Jeffrey Giansiracusa and Gregory Sankaran, using topological methods.

**Sarah Koch, Harvard University, An analytic construction of the Deligne-Mumford compactification of the moduli space of curves**

We outline a proof that the Deligne-Mumford compactification of the moduli space of curves is isomorphic (as an analytic space) to the quotient of augmented Teichmüller space by the action of the mapping class group. The main difficulty is putting a complex structure on this quotient. This is joint work with John H. Hubbard.

**Eduard Looijenga, University of Utrecht, Cohomological amplitude of moduli spaces of curves**

We show that the cohomological amplitude of the universal curve of genus $g > 0$ is at most $g - 1$. This implies the theorems of Harer on the homotopy of the moduli spaces of curves as well as Diaz’s theorem which states that any complete subvariety of $M_g, g > 1$ can have dimension at most $g - 2$.

**Howard Masur, University of Chicago, Winning sets for Schmidt games, badly approximable real numbers, rational billiards and geodesics in moduli space**

In the 1970’s Schmidt introduced a game to be played between two players in Euclidean space. Winning sets for these games have nice properties. The main example Schmidt considered were ”Diophantine” reals that are badly approximated by rationals, or equivalently, flow lines on a torus badly approximated by closed curves. It is classical that these reals correspond to geodesics that stay in a compact set in the moduli space of lattices. They also have an interpretation in terms of orbits in a square billiard table. In joint work with Jon Chaika and Yitwah Cheung we consider general rational billiards and the corresponding translation surfaces. There is a corresponding notion of a Diophantine flow direction on the surface. It is one that is badly approximated by saddle connections. We show that the set of badly approximated flow directions form a winning set for the Schmidt game.

I will discuss all of these ideas: Schmidt’s game, the basic example of Diophantine reals, the connection with billiards in a square before considering more general rational billiards.

**Poster Session**

- Derek Allums (Rice University) and J. M. Landsberg (Texas A&M University), *Border rank of ternary trilinear forms and the $j$-invariant*

  We first describe how one associates a cubic curve to a given ternary trilinear form. We explore relations between the rank and border rank of the form and the geometry of the corresponding cubic curve. When the curve is smooth, we show there is no relation. When the curve is singular, normal forms are available, and we review the explicit correspondence between the normal forms, rank and border rank.

- Khek Lun Harold Chao (Indiana University), *CAT(0) spaces with boundary the join of two Cantor sets*

  We will show that if a proper complete CAT(0) space $X$ has a visual boundary homeomorphic to the join of two Cantor sets, and $X$ admits a geometric group action by a group containing a subgroup
isomorphic to $\mathbb{Z}^2$, then its Tits boundary is the spherical join of two uncountable discrete sets. If $X$ is geodesically complete, then $X$ is a product, and the group has a finite index subgroup isomorphic to a lattice in the product of two isometry groups of bounded valence bushy trees.

- Hyunjoo Cho, Sema Salur, and Firat Arikan (University of Rochester), *Existence of almost contact structures on manifolds with $G_2$-structures*

  We show the existence (co-oriented) contact structures on certain classes of $G_2$-manifolds, and that these two structures are compatible in certain ways. We also prove that any seven-manifold with a spin structure has an almost contact structure, and construct explicit almost contact structures on manifolds with $G_2$-structures. Moreover, we also show that one can extend any almost contact structure on an associative submanifold to whole $G_2$-manifold.

- Thanos Gentimis and Maria Bampasidou (University of Florida), *An algebraic topology model for research and development*

  In this paper we describe a model based on basic algebraic topology that describes interactions between companies in terms of research and development. We then apply our model to collaborations between mathematicians in the creation of mathematical papers.

- Sergii Kutsak (University of Florida), *Essential manifolds with extra structures*

  I consider classes of algebraic manifolds $\mathcal{A}$, of symplectic manifolds $\mathcal{S}$, of symplectic manifolds with the hard Lefschetz property $\mathcal{H}S$, and the class $\mathcal{CS}$ of cohomologically symplectic manifolds. For every class of manifolds $\mathcal{C}$, I denote by $\mathcal{EC}(\pi, 2n)$ a subclass of $n$-dimensional rationally essential manifolds with fundamental group $\pi$. I will prove that for all the above classes with symplectically aspherical form the condition $\mathcal{EC}(\pi, 2n) \neq 0$ implies that $\mathcal{EC}(\pi, 2n - 2) \neq 0$ for every $n > 2$. Also I will show that all the inclusions $\mathcal{EA} \subset \mathcal{ECHS} \subset \mathcal{EE} \subset \mathcal{EC}\mathcal{S}$ are proper.

- Dmitry Zakharov and Samuel Grushevsky (Stony Brook University), *Partial compactification of the zero section of the universal abelian variety*

  The moduli space of principally polarized abelian varieties is one of the central objects of study in algebraic geometry. The moduli space is not compact, and admits several natural compactifications. All of these compactifications are extensions of Mumford’s partial compactification by semi-abelic varieties of torus rank one. The partial compactification is the base for a universal family that admits a zero-section. In our joint work with Samuel Grushevsky, we calculate the class of the zero section in the Chow ring of the partial compactification of the universal abelian variety.

- Letao Zhang (Rice University), *Representation theory and Hilbert schemes of points on K3 surfaces.*

  Let $X$ denote a general deformation of the Hilbert schemes of $n$ points on $K3$ surfaces, and $G_x$ be the associated group acting on the cohomology ring. We computed the graded character formula associated to the $G_x$ action on the cohomology ring of $X$. Also, we could use the formula to deduct the generating series of the number of canonical Hodge classes for each $n$. 