Assume the axioms of Euclidean geometry, as in Chapter 5.

1. Suppose $\triangle ABC$ and $\triangle DEF$ are triangles for which $\angle BAC \cong \angle EDF$ and $\angle ABC \cong \angle DEF$.

(a) Is it necessarily true that $\triangle ABC \sim \triangle DEF$? Explain.

Yes: $\mu(\angle BAC) + \mu(\angle ABC) + \mu(\angle ACB) = 180$ and
$\mu(\angle EDF) + \mu(\angle DEF) + \mu(\angle FDE) = 180$.

Since $\mu(\angle BAC) = \mu(\angle EDF)$ and $\mu(\angle ABC) = \mu(\angle DEF)$, it follows that $\mu(\angle ACB) = \mu(\angle FDE)$, so $\triangle ABC \sim \triangle DEF$.

(b) Is it necessarily true that $\triangle ABC \cong \triangle DEF$? Explain.

No: The triangles are similar, but need not be congruent.

For example, $\triangle ABC$ and $\triangle DEF$ could both be right triangles, with $AB = BC = CA = 1$ and $DE = EF = FA = 1$. 
2. Let $\triangle ABC$ be a 30-60-90-triangle (that is, a triangle whose interior angles measure $30^\circ, 60^\circ, 90^\circ$). Prove that in $\triangle ABC$, the length of the side opposite the $30^\circ$ angle is one half the length of the hypotenuse. (Hint: Use the Angle Construction and Point Construction Postulates to construct a triangle congruent to $\triangle ABC$ that shares a side with $\triangle ABC$ and so that the two together form an equilateral triangle.)

Suppose $\mu(\angle AEB) = 30^\circ$ and $\mu(\angle DAE) = 90^\circ$.

By the Angle Construction Postulate, there is a ray $\overrightarrow{CD}$ with $	ext{B}$ on the opposite side of $\overline{AC}$ or $\overline{E}$, for which $\mu(\angle ACD) = 30^\circ$. By the Point Construction Postulate, there is a point $D'$ on $\overrightarrow{CD}$ such that $CD' = BC$. Since $CD' = BC$, $\triangle ABC$ is isosceles, so $\angle BAC \cong \angle A'AC$.

By the ASA Theorem, $\triangle BAC \cong \triangle D'AC$.

Therefore, $BA = AD'$. Since the sum of the angles in $\triangle ABC$ is $180^\circ$, each of angles $\angle ABC$ and $\angle ABD'$ must measure $60^\circ$. So $\triangle ABD'$ is equilateral, and therefore equilateral.

It follows that $BA = \frac{1}{2} BC$. 