1. Let \( D \) denote the *square metric* on the Cartesian plane \( \mathbb{R}^2 \), that is,
\[
D((x_1, y_1), (x_2, y_2)) = \max\{|x_2 - x_1|, |y_2 - y_1|\}
\]
for all real numbers \( x_1, x_2, y_1, y_2 \). Consider the model in which points, lines, half-planes, and angle measure are as usual for the Cartesian plane, but distance is given by the square metric. Let \( A = (0, 0), B = (2, 0), \) and \( C = (2, 2) \). Let \( A' = (-4, 2), B' = (-2, 0), \) and \( C' = (0, 2) \).

(a) Sketch the two triangles \( \triangle ABC \) and \( \triangle A'B'C' \).

(b) Find all angle measures in each triangle, and record them.
\[
\begin{align*}
\angle ABC &= 90^\circ & \angle BCA &= 45^\circ & \angle CAB &= 45^\circ \\
\angle A'B'C' &= 90^\circ & \angle B'C'A' &= 45^\circ & \angle C'A'B' &= 45^\circ
\end{align*}
\]

(c) Find all side lengths for each triangle, and record them. (Remember to use the square metric!)
\[
AB = 2 & \quad BC = 2 & \quad AC = 2 \\
A'B' = 2 & \quad B'C' = 2 & \quad A'C' = 4
\]

(d) Does the Side-Angle-Side Postulate hold for this model? Explain.

No: \( \overline{AB} \neq \overline{A'B'}, \overline{BC} \neq \overline{B'C'}, \) and \( \angle ABC \neq \angle A'B'C' \), however, if the SAS Postulate holds, then \( \triangle ABC \cong \triangle A'B'C' \), however these two triangles are not congruent since \( \overline{AC} \neq \overline{A'C'} \). Therefore the SAS Postulate does not hold.
2. Prove Theorem 3.5.9: If \( \ell \) is a line and \( P \) is a point on \( \ell \), then there exists exactly one line \( m \) such that \( P \) lies on \( m \) and \( m \perp \ell \). (Hint: Use the Angle Construction Postulate.)

Let \( H \) be one of the half-planes bounded by \( \ell \).
Let \( E \) be a point on \( \ell \), and let \( P \) be a point in the half-plane \( H \).
By the Angle Construction Postulate, there is a unique ray \( \overrightarrow{PE} \) such that \( E \in m \) and \( m \perp \ell \).

3. Prove the Vertical Angles Theorem (Theorem 3.5.13): If angles \( \angle BAC \) and \( \angle DAE \) form a vertical pair, then \( \angle BAC \cong \angle DAE \). (Hint: Use the Linear Pair Theorem.)

By the definition of vertical pair, \( \angle BAC \) and \( \angle EAC \) form a linear pair, and \( \angle EAC \) and \( \angle DAE \) form a linear pair.

By the Linear Pair Theorem,

\[
\angle BAC + \angle EAC = 180^\circ \quad \text{and} \quad \angle EAC + \angle DAE = 180^\circ.
\]

So, \( \angle BAC = 180^\circ - \angle EAC \)
\[
= 180^\circ - \left( 180^\circ - \angle DAE \right)
\]
\[
= \angle DAE.
\]

So, \( \angle BAC \cong \angle DAE \).