1. \((\Rightarrow)\) Assume the interior angles of \(\triangle ABC\) measure 45°, 45°, and 90°. Then \(\triangle ABC\) is a right triangle by definition. By the Converse to the Isosceles Triangle Theorem, the sides opposite the two 45° angles are congruent, and so \(\triangle ABC\) is isosceles.

\((\Leftarrow)\) Assume \(\triangle ABC\) is right and isosceles. Since one of the angles is 90°, the other two must be acute and congruent by the Isosceles Triangle Theorem.

Since the sum of all angles measures 180°, the sum of measures of the two acute angles is 180° - 90° = 90°, and since they are congruent, each must be 45°.

2. Since \(\triangle ABC \sim \triangle DEF\), all corresponding angles are congruent, and in particular, 

\[ \angle BAC = \angle DEF \text{ and } \angle ABC = \angle DEF. \]

Since \(AB \sim DE\), by the SAS Postulate, \(\triangle ABC \cong \triangle DEF\).

3. (a) 

(b) Take \(\triangle A'B'C'\) to be the same as \(\triangle ABC\) above, and take \(\triangle A'B'E'F'\) to be similar to \(\triangle DEF\) but with all side lengths half those of \(\triangle DEF\):

4. First note that by the SSS Theorem, \(\triangle ABD \cong \triangle CBD\), and both triangles are isosceles. So in particular,

\[ \angle ABD = \angle CBD. \]

Now put in the other diagonal: Since \(\overline{BE} \cong \overline{BE}\), by the SAS Postulate,

\[ \triangle ABE \cong \triangle CBE. \]

This implies \(\overline{AE} = \overline{EC}\), so \(E\) is the midpoint of \(\overline{AC}\). Similarly, \(E\) is the midpoint of \(\overline{BD}\).
5. This can be proven similarly to Exam 2 #9: By the SAS Postulate, \( \triangle ABE \cong \triangle CBE \) (since \( BE = BE \)). So \( \angle AEB \cong \angle CEB \).

Since these two angles form a linear pair, they are also supplementary, and consequently each is a right angle.

6. By the reasoning in #4, \( \triangle ABD \cong \triangle CBD \), and both triangles are isosceles, so we have four congruent angles: \( \angle ABD, \angle ADB, \angle DBC, \angle BDC \).

In particular, \( \angle ABD \) and \( \angle BDC \) are congruent alternate interior angles with respect to the lines \( AB \) and \( BC \). By the Alternate Interior Angles Theorem, \( AB \parallel BC \). Similarly, \( \overline{AB} \parallel \overline{BC} \). So \( \square ABCD \) is a parallelogram.