MATH 367 HOMEWORK ASSIGNMENT 7 SOLUTIONS

1. \( \angle 5 \equiv \angle 2 \) (Corresponding angles)
   \( \angle 4 \equiv \angle 1 \) (Vertical angles)
   \[ x + y + z = 180 \]
   \[ y + 2y + 72 = 180 \]
   \[ 3y = 90 \]
   \[ y = 30 \text{ and } x = 60 \]

2. Consider the exterior angle \( \angle BCD \). Since \( \angle BCD \) and \( \angle ACB \) form a linear pair, \( m(\angle BCD) + m(\angle ACB) = 180 \).
   Also the sum of the interior angles of the triangle is:
   \[ m(\angle ABE) + m(\angle BCB) + m(\angle CEB) = 180 \]
   Solving for \( m(\angle ABE) \) in the first equation, we have \( m(\angle ABE) = 180 - m(\angle BCD) \).
   Substituting into the second equation, we have:
   \[ m(\angle ABE) + m(\angle BCD) + m(\angle CEB) = 180 \]
   Solving for \( m(\angle BCD) \), we have \( m(\angle BCD) = m(\angle ABE) - m(\angle CEB) \).

3. By Thm 5.1.1, alternate interior angles are congruent,
   \( \theta \parallel \theta' \)

4. Let \( \square ABCD \) be a parallelogram and consider the diagonal \( \overline{AC} \).
   Then \( \angle ACD \) and \( \angle BAC \) are alternate interior angles with respect to parallel lines \( \overline{AB} \) and \( \overline{CD} \), so they are congruent.
   Also \( \angle DAC \) and \( \angle CBA \) are congruent, by similar reasoning. Since \( \overline{AC} \parallel \overline{CD} \), by the ASA Theorem, \( \triangle ABC \cong \triangle CDA \).
   A similar argument shows that \( \overline{BB} \) divides the parallelogram into two congruent triangles.

5. By #4, \( \overline{AB} \equiv \overline{CD} \) and \( \overline{BC} \equiv \overline{DA} \).

6. By #4, \( \angle ABC \equiv \angle CDA \) and \( \angle BAD \equiv \angle CDB \).

7. Note that \( \overline{AB} \) is a transversal for \( \overline{BC} \) and \( \overline{AB} \), and all interior angles are right angles. By the Alternate Interior Angles Theorem, \( \overline{BC} \parallel \overline{AB} \). Similarly, \( \overline{AB} \parallel \overline{CD} \).
   So \( \square ABCD \) is a parallelogram.