Corollary 61. Assume a transversal line intersects two lines. Then the two lines are parallel if and only if the corresponding angles are congruent.

Proof: Let \( l \) be a transversal of lines \( m, n \).

(\( \Rightarrow \)) Assume \( m \) and \( n \) are parallel.
By Thm 60, alternate interior angles are congruent: \( \angle 2 \cong \angle 3 \).
By Cor 38, \( \angle 1 \cong \angle 3 \) since they are vertical angles. Therefore \( \angle 1 \cong \angle 2 \), that is, corresponding angles are congruent.

(\( \Leftarrow \)) Assume corresponding angles are congruent: \( \angle 1 \cong \angle 2 \).
By Cor 41, lines \( m \) and \( n \) are parallel. \( \square \)

Corollary 62. Opposite sides of a rectangle are congruent.

Proof: Let \( \square ABCD \) be a rectangle.
By definition of a rectangle, the four sides form four right angles.
By definition, a right angle is congruent to its supplement.
Consider lines \( BC \) and \( AD \), with transversal \( AB \): Alternate interior angles are congruent (they are right angles), so by Thm 60, \( BC \) and \( AD \) are parallel.
Now consider \( BD \) to be a transversal of \( BC \) and \( AD \). By Thm 60, alternate interior angles are congruent, so \( \angle CBD \cong \angle ADB \).
By similar arguments to the above, \( AB \) and \( CD \) are parallel, and \( \angle ABD \cong \angle CDB \).
Clearly \( BD \cong BD \), and so by Thm 17 (ASA), \( \triangle ABD \cong \triangle CDB \).
By CPCFC, \( AB \cong CD \) and \( BC \cong AD \). \( \square \)

Problem 76. Answers will vary somewhat. I counted 169 squares, so \( A(X) = 169 \) square units (or 16,900 square miles since each unit square has area \( 10 \times 10 = 100 \) square miles).

Problem 77. \( A(Y) = 289 \) square units (or 28,900 square miles).

Problem 78. (i) \( 169 \leq A(WV) \leq 289 \) (or 16,900 sq. mile, \( \leq A(WV) \leq 28,900 \text{ sq. mile} \))
(ii) \( \frac{1}{2}(16,900 + 28,900) = 22,900 \) square miles
(iii) 234 unit squares, or 23,400 square miles
(iv) Encyclopedia Britannica (online): 24,200 square miles
Estimate (ii) is closer.

Problem 79. Choose a smaller unit, e.g. squares whose side lengths are 1 mile instead of 10 miles.