Math 365 Partial solutions to Exam 2

1. (a) No. Counterexample: n=6. (There are many other counterexamples possible.)
(b) If 12|n, then 2|n and 6|n. True.

2. (a) 5 \cdot 4 \cdot 3 \cdot 2 = 120
(b) 3 \cdot 2 \cdot 3 \cdot 2 = 36

3. For this problem, a Venn diagram is very helpful, and alternative methods of solution can be based on a diagram. The solution here is based on known relations among cardinalities of sets.
(a) Let A denote the set of survey respondents owning an American car, and let J denote the set of those owning a Japanese car. Then \( n(A \cup J) = 110 - 12 = 98 \), and so
\[
\begin{align*}
  n(A \cup J) &= n(A) + n(J) - n(A \cap J) \\
  98 &= 73 + 39 - n(A \cap J)
\end{align*}
\]
and it follows that \( n(A \cap J) = 14 \).
(b) Let G denote the set of survey respondents owning a German car. From the given information, we have \( n(A \cup J \cup G) = 107 \), \( n(A \cap G) = 8 \), and \( n(J \cap G) = 0 \), and from this it also follows that \( n(A \cap J \cap G) = 0 \). So
\[
\begin{align*}
  n(A \cup J \cup G) &= n(A) + n(J) + n(G) - n(A \cap J) - n(A \cap G) - n(J \cap G) + n(A \cap J \cap G) \\
  107 &= 73 + 39 + n(G) - 14 - 8 - 0 + 0
\end{align*}
\]
and from this it follows that \( n(G) = 17 \).

4. (a) Same: \( 224 \cdot 5 = 112 \cdot 2 \cdot 5 = 112 \cdot 10 \)
(b) Not the same: \( 14,800 - 99 = (14,800 + 1) - (99 + 1) = 14,801 - 100 \), which is not the same as \( 14,799 - 100 \) (in each expression, 100 is subtracted from a number, but those numbers are different).

5. 
\[
\begin{array}{ccc}
  3 & 2,5,8 & 4 & 0,2,4,6,8 \\
  9 & 5 & 11 & 7
\end{array}
\]
6.

97 is prime (check it is not divisible by 2, 3, 5, 7, and this suffices, as \(\sqrt{97} < \sqrt{100} = 10\), and these are all the prime numbers less than 10)

187 is composite (as it is divisible by 11)

\(2^7 - 1\) is prime (it is equal to 127, and again one can check divisibility by primes less than or equal to \(\sqrt{127} < \sqrt{144} = 12\))

\(19 \cdot 23 + 23 \cdot 89\) is composite (since 23 is a factor of each term, it is a factor of the number, specifically, the number is equal to \(23 \cdot (19 + 89) = 23 \cdot 108\))

\(19! + 17\) is composite (Since \(19! = 19 \cdot 18 \cdot 17 \cdot 16 \cdots 3 \cdot 2 \cdot 1\), the number 17 is a factor of each term of the number, specifically, the number is equal to \(17 \cdot (19 \cdot 18 \cdot 16 \cdots 3 \cdot 2 \cdot 1 + 1)\))

7. (a)

\[
\begin{align*}
91 &= 35 \cdot 2 + 21 \\
35 &= 21 \cdot 1 + 14 \\
21 &= 14 \cdot 1 + 7 \\
14 &= 7 \cdot 2
\end{align*}
\]

So GCD(91, 35) = 7 (the last nonzero remainder above)

(b) LCM(91, 35) = \(\frac{91 \cdot 35}{7} = \frac{91 \cdot 5 \cdot 7}{7} = 91 \cdot 5 = 455\)

8. (a) True

(b) False. Counterexample: \(A = \{1, 2, 3\}, \ B = \{3, 4, 5\}, \ C = \{3, 4\}\). Then \(A \cap B = \{3\} = A \cap C\), but \(B \neq C\). (Many other counterexamples are possible.)

(c) False. Counterexample: \(n = 21\) (\(n\) is divisible by 3, but each digit of \(n\) is not)

(d) False. Counterexample: \(a = 2, b = 4, d = 8\) (then \(d\) divides \(2 \cdot 4 = 8\) but \(d\) does not divide 2 nor 4)

(e) True.