There are 7 questions, for a total of 100 points. Point values are written beside each question. *No calculators allowed. Show your work for full credit.*

1. [8 points] (a) Order the following Egyptian numerals from least to greatest:

\[ \underline{\underline{\text{IIII}}}, \quad \underline{\underline{\text{III}}} \quad \underline{\underline{\text{IIIIII}}} \]

(b) *Without* converting to decimals, add the following two Egyptian numerals. Briefly *explain* how you obtained your answer.

\[ \underline{\underline{\text{IIII}}}, \quad \underline{\underline{\text{IIIIII}}} \]

2. [16] (a) Each of the following illustrates a property of addition and/or multiplication of whole numbers. Identify the property illustrated.

\[ (2 + 3) + 5 = 2 + (3 + 5) \]

\[ 28 \cdot 1 = 28 = 1 \cdot 28 \]

\[ 4 \cdot (10 + 2) = 4 \cdot 10 + 4 \cdot 2 \]

(b) Identify *two* properties illustrated by the following.

\[ 2 \cdot (19 \cdot 50) = (2 \cdot 50) \cdot 19 \]
3. [20] Perform each of the operations in base 5, \textit{without} converting to base 10. Show your work. (If you wish to convert to base 10, to check your answers only, that is fine.)

(a) \(11_{\text{five}} + 24_{\text{five}}\)  
(b) \(32_{\text{five}} - 13_{\text{five}}\)

(c) \(12_{\text{five}} \cdot 23_{\text{five}}\)  
(d) \(41_{\text{five}} \div 3_{\text{five}}\)
4. [12] Vera calculated the following. Redo the calculation correctly, and write a sentence explaining to Vera where she made a mistake, and how to fix her calculation.

\[
\begin{array}{c}
243 \\
-138 \\
\end{array}
\]

5. [16] In each of the following, identify the sequence as arithmetic or geometric, and find the \( n \)th term.
(a) 5, 11, 17, 23, 29, . . .

(b) 2, 6, 18, 54, 162, . . .
6. [12] Without computing each sum, find which is greater, $S$ or $T$, and by how much.

$$S = 3 + 6 + 9 + 12 + \cdots + 99$$
$$T = 5 + 8 + 11 + 14 + \cdots + 101$$

7. [16] (True/False and Counterexample.) For each of the following statements, write “T” if it is true and “F” if it is false. If it is false, give a counterexample.

(a) ________ For all whole numbers $n$, \[ \frac{n + 3}{3} = n. \]

(b) ________ For all whole numbers $n$, \[ n + 3 = 3 + n. \]

(c) ________ For all whole numbers $n$, \[ (n + 1)^3 = n^3 + 1^3. \]

(d) ________ For all whole numbers $n$, \[ \frac{3n}{3} = n. \]