Math 220  Exam 1
September 28, 2012
S. Witherspoon

Name

There are 5 questions, for a total of 100 points. Point values are written beside each question.

1. Consider the statement: For all integers $m$ and $n$, if $mn$ is even, then $m$ is even or $n$ is even.
   (a) [5 points] Write the converse of this statement.
   (b) [5] Write the contrapositive of this statement.
   (c) [5] Write the negation of this statement.
   (d) [5] Which of the above four statements (the proposition, its converse (a), its contrapositive (b), its negation (c)) are true? (You need not justify your answer.)
2. [15] Prove that $\sqrt{2}$ is irrational.
3. Let $A = \{1, 2, 3, 5, 8\}$ and $B = \{2, 4, 6, 8\}$. Find the following sets:

(a) $A \cap B$

(b) $A \cup B$

(c) $A - B$

(d) $B - A$

(e) the power set, $\mathcal{P}(A \cap B)$, of the intersection $A \cap B$
4. Consider the following two sets:
\[ A = \{ n \in \mathbb{Z} \mid n = 3s + 1 \text{ for some } s \in \mathbb{Z} \} \]
\[ B = \{ n \in \mathbb{Z} \mid n = 9t + 4 \text{ for some } t \in \mathbb{Z} \} \]

(a) [5] List at least 5 elements of \( A \) and at least 5 elements of \( B \).

(b) [10] Is \( A \subseteq B \)? Prove or disprove.

(c) [10] Is \( B \subseteq A \)? Prove or disprove.
5. (a) [5] Let \( B = \{2, 3, 5, 8\} \) and \( C = \{3, 7\} \). Find \( B \times C \) (that is, write out all the elements of this set).

(b) [10] Prove that for all sets \( A, B, \) and \( C \), if \( A \subseteq B \), then \( A \times C \subseteq B \times C \).