1. (a) For all integers $m$ and $n$, if $m + n$ is divisible by 4, then $m$ is even and $n$ is even.

(b) For all integers $m$ and $n$, if $m + n$ is not divisible by 4, then $m$ is odd or $n$ is odd.

(c) There exist integers $m$ and $n$ such that $m$ is even and $n$ is even and $m + n$ is not divisible by 4.

(d) negation

2. Let $m$ and $n$ be odd integers. Then $m = 2s + 1$ and $n = 2t + 1$ for some integers $s$ and $t$. It follows that

$$m + n = (2s + 1) + (2t + 1) = 2(s + t + 1),$$

which is even since $s + t + 1$ is an integer.

3. (a) $\{2\}$  (b) $\{0, 1, 2, 4\}$  (c) $\{0, 1\}$  (d) $\{4\}$  
   (e) $\{(0, 2), (0, 4), (1, 2), (1, 4), (2, 2), (2, 4)\}$

4. (a) No. For example, $5 \in A$ and $5 \notin B$. (There are many other possibilities as well.)

(b) Yes. Let $n \in B$, so that $n = 4t - 1$ for some integer $t$. Rewriting, we have

$$n = 4t - 2 + 1 = 2(2t - 1) + 1,$$

which is an element of $A$. (Explicitly, we may let $s = 2t - 1$ to obtain $n = 2s + 1$ for the integer $s$.) Therefore $B \subseteq A$.

5. (a) No. For example, $f(-1) = f(1)$ and yet $-1 \neq 1$.

(b) No. For example, 0 is not in the image of $f$.

(c) $[1, 2]$  

(d) $[-1, 1]$