1. (a) \[
\lim_{x \to 0} \frac{\sin(2x)}{\sin(x)} = \lim_{x \to 0} \frac{2 \sin(x) \cos(x)}{\sin(x)} = \lim_{x \to 0} (2 \cos(x)) = 2
\]
(b) \[
\lim_{x \to \infty} \frac{2e^{2x}}{2e^x + e^x} = \lim_{x \to \infty} \frac{2}{e^x + 1} = \frac{2}{0+1} = 2
\]
(c) \[
\lim_{x \to 1} \left( \log_2(x^2 - 1) - \log_2(x-1) \right) = \lim_{x \to 1} \log_2 \left( \frac{x^2 - 1}{x-1} \right) = \lim_{x \to 1} \log_2 (x+1) = \log_2 (2) = 2
\]

2. (a) \[
f'(x) = \frac{(x^2+1)(2x) - x^2(2x)}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}
\]
(b) \[
f'(x) = 2 \sec(xe^x) \cdot \left( \sec(xe^x) \right)'
= 2 \sec(xe^x) \cdot \left( \sec(xe^x) \tan(xe^x) \right) \cdot (xe^x)'
= 2 \sec(xe^x) \sec(xe^x) \tan(xe^x) (xe^x + e^x)
= 2xe^x(x+1) \sec^2(xe^x) \tan(xe^x)
\]

3. First consider \(g(x) = \tan x\), whose derivative is \(g'(x) = \sec^2 x\). When \(g(x) = 0\), i.e., when \(x = k\pi\) (\(k \in \mathbb{Z}\)), \(g'(x) = \sec^2(k \pi) = (\sec(k \pi))^2 = (\pm 1)^2 = 1\). This fact and examination of the graph show that \(f(x) = |\tan x|\) is not differentiable at those values. It is also not differentiable where \(t\) is not defined, e.g., when \(x = \pm \frac{\pi}{2} + 2k\pi\) (\(k \in \mathbb{Z}\)). Combining these two expressions, we see that \(f\) is not differentiable at \(x = \pm \frac{\pi}{2} + 2k\pi\) (\(k \in \mathbb{Z}\)).

4. Note that the given point \((1, 0)\) results from taking \(t = 0\).
\[r'(t) = \langle e^t, 2 \cos t \rangle\] \(\Rightarrow r'(0) = \langle 1, 2 \rangle\). The slope is \(\frac{2}{1} = 2\).
The tangent line is \(y = 0 = 2(x-1)\), or \(y = 2x-2\).

5. Note that \(g'(3) = 0\) since \(f(0) = 3\), and \(f'(x) = 1 + \frac{\pi}{2} \sec \left( \frac{\pi}{2} x \right)\).
\[
g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(0)} = \frac{1}{1 + \frac{\pi}{2} \sec(0)} = \frac{1}{1 + \frac{\pi}{2}} = \frac{2}{2+\pi}
\]

6. \[
\frac{dx}{dt} = -\frac{1}{4} \text{ rad}/h
\]
\[
\tan \theta = \frac{900}{x}
\]
\[
\sec^2 \theta \frac{d\theta}{dt} = -\frac{900}{x^2} \frac{dx}{dt}
\]
\[
\frac{4}{3} \cdot \left( \frac{1}{4} \right) = -\frac{900}{900 - 3} \cdot \frac{dx}{dt}
\]
\[
\frac{dx}{dt} = \frac{900}{900 - 3} \text{ ft}/h
\]

When \(\theta = \frac{\pi}{6}\): \[\tan \frac{\pi}{6} = \frac{900}{x}\]
\[
\frac{x}{\sqrt{3}} = \frac{900}{x}
\]
\[
x = 900\sqrt{3}
\]
\[
\sec^2 \frac{\pi}{6} = \frac{1}{\cos^2 \frac{\pi}{6}} = \frac{1}{\left( \frac{\sqrt{3}}{2} \right)^2} = \frac{4}{3}
\]
7. \( y = \frac{c}{x} \)

\[ y' = -\frac{c}{x^2}, \]

at a point \((x_0, y_0)\) for which \(x_0y_0 = c\),

the tangent line therefore has slope \( m = -\frac{c}{x_0^2} \)

and equation \( y - y_0 = -\frac{c}{x_0^2}(x - x_0) \)

\[ y = -\frac{c}{x_0^2}x + \frac{c}{x_0} + y_0 = -\frac{c}{x_0^2}x + 2y_0 \quad \text{(since } y_0 = \frac{c}{x_0} \text{)} \]

The y-intercept is \(2y_0\).

By symmetry, the x-intercept is \(2x_0\).

Their product is \((2y_0)(2x_0) = 4x_0y_0 = 4c \quad \text{(since } x_0y_0 = c \text{)}\)