1.3. We have \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \). Since \( P(A \cup B) \leq 1 \), this implies that \( P(A \cap B) \geq P(A) + P(B) - 1 = 0.1 \). We also have \( P(A \cap B) \leq \min(P(A), P(B)) = 0.4 \). It is easy to see that 0.1 and 0.4 are the minimal and the maximal possible value of \( P(A \cap B) \). The first case happens when \( P(A \cup B) = 1 \), the second when \( A \subset B \).

1.5. First solution. The probability that the first player wins on first turn is \( \frac{m}{m+n} \). The probability of winning for the first player on the second turn is \( \frac{n}{m+n} \cdot \frac{n-1}{m+n-1} \cdot \frac{n-2}{m+n-2} \cdot \frac{n-3}{m+n-3} \cdot \frac{m}{m+n-4} \). For the third turn it is \( \frac{n}{m+n-1} \cdot \frac{n-1}{m+n-2} \cdot \frac{n-2}{m+n-3} \cdot \frac{n-3}{m+n-4} \).

Second, recursive solution. Let \( p_{m,n} \) be the probability that the first player wins. Then \( p_{m,n} \) is equal to \( \frac{m}{m+n} + \frac{n}{m+n} \cdot p \), where \( p \) is the conditional probability that the first player wins after removing a black ball. But this probability is the same as the probability that the player starting the game for \( m \) white and \( n-1 \) black balls loses, so

\[
 p_{m,n} = \frac{m}{m+n} + \frac{n}{m+n} (1 - p_{m,n-1}),
\]

or

\[
 p_{m,n} = 1 - \frac{n}{m+n} p_{m,n-1},
\]

which gives a recursive formula for \( p_{m,n} \) (using the fact that \( p_{m,0} = 1 \)).

1.6. Let them toss at first \( n \) coins each. Let \( p \) be the probability that then Alice has more “heads”. Then \( p \) is also the probability that Bob has more “heads”. The probability that they have equal number of heads is \( 1 - 2p \). In the first case (when Alice has more “heads”), Bob can not get more “heads” than Alice after tossing the remaining one coin. In the second case Bob will have more “heads” independently of the result of the last toss. In the third case (when they have the same number of “heands” after tossing \( n \) coins each) the probability that Bob has more “heads” is \( 1/2 \). It follows that the probability that Bob will get more “heads” is \( p + (1 - 2p) \frac{1}{2} = \frac{1}{2} \).

1.7. \( P(A) = 1/13 \). \( P(B) = 1/4 \). Probability that the first card is an ace of spades, and the second is a spade is \( \frac{12}{52} \cdot \frac{13}{51} \). Probability that the first card is a non-spade ace, and the second is a spade is \( \frac{3 \cdot 13}{52} \cdot \frac{12}{51} \). It follows that probability that the first card is an ace and the second is a spade is

\[
 \frac{12 + 3 \cdot 13}{52 \cdot 51} = \frac{51}{52 \cdot 51} = \frac{1}{13} \cdot \frac{1}{4},
\]

hence the events are independent.