Math 411 — Problem Set 1
Issued: 06.01     Due: 06.08

1.1. A six-sided die is loaded in a way that each even face is twice as likely as each odd face. All even faces are equally likely, as are all odd faces. Construct a probabilistic model for a single roll of this die and find the probability that the outcome is less than 4.

1.2. Two dice are rolled. What is the probability that (a) the two numbers will differ by 1 or less and (b) the maximum of the two numbers will be five or larger?

1.3. Given two events A and B with \( P(A) = 0.4 \) and \( P(B) = 0.7 \), what are the maximum and minimum possible values for \( P(A \cap B) \)?

1.4. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely. (a) Find the probability that doubles are rolled. (b) Given that the roll results in a sum of 4 or less, find the conditional probability that doubles are rolled. (c) Find the probability that at least one die roll is a 6.

1.5. Two players take turns removing a ball from a jar that initially contains \( m \) white and \( n \) black balls. The first player to remove a white ball wins. Find the probability that the starting player wins.

1.6. Alice and Bob have \( 2n+1 \) coins, each coin with probability of heads equal to 1/2. Bob tosses \( n+1 \) coins, while Alice tosses the remaining \( n \) coins. Assuming independent coin tosses, show that the probability that after all coins have been tossed, Bob will have gotten more heads than Alice is 1/2.

1.7. Suppose that we draw 2 cards out of a deck of 52. Let \( A = \text{“the first card is an ace”} \) and \( B = \text{“the second card is a spade”} \). Are \( A \) and \( B \) independent?
1.8. A family has 3 children, each of whom is a boy or a girl with probability $1/2$. Let $A$ = “there is at most 1 girl” and $B$ = “the family has children of both sexes”. (a) Are $A$ and $B$ independent? (b) Are $A$ and $B$ independent if the family has 4 children?

1.9. Let $A$ and $B$ be two independent events with $P(A) = 0.4$ and $P(A \cup B) = 0.64$. What is $P(B)$?

1.10. Three independent events have probabilities $1/4$, $1/3$, and $1/2$. What is the probability that exactly one will occur?