MATH 411 Homework 1

1.1. Two dice are rolled. What is the probability that (a) the two numbers will differ by 1 or less and (b) the maximum of the two numbers will be five or larger?

1.2. Given two events $A$ and $B$ with $P(A) = 0.4$ and $P(B) = 0.7$, what are the maximum and minimum possible values for $P(A \cap B)$?

1.3. Suppose we roll a red and a green die. Let $A =$”the red die shows a 2 or a 5” and $B =$”the sum of the two dice is at least 7.” Are $A$ and $B$ independent?

1.4. A family has 3 children, each of whom is a boy or a girl with probability 1/2. Let $A =$ “there is at most 1 girl” and $B =$ “the family has children of both sexes”. (a) Are $A$ and $B$ independent? (b) Are $A$ and $B$ independent if the family has 4 children?

1.5. Let $A$ and $B$ be two independent events with $P(A) = 0.4$ and $P(A \cup B) = 0.64$. What is $P(B)$?

1.6. Three independent events have probabilities 1/4, 1/3, and 1/2. What is the probability that exactly one will occur?

1.7. Suppose we roll two dice and let $X$ and $Y$ be the two numbers that appear. Find the distribution of $|X - Y|$.

1.8. Five people play a game of “odd man out” to determine who will pay for the pizza they ordered. Each flips a coin. If only one person gets heads (or tails) while the other four get tails (or heads) the he is the odd man and has to pay. Otherwise they flip again. What is the expected number of tosses needed to determine who will pay?

1.9. Suppose we pick a month at random from a non-leap year calendar and let $X$ be the number of days in the month. Find the mean and variance of $X$. 