1a) Find the order of $9 \pmod{101}$.
(3 pt)
1b) Find the order of $2 \pmod{101}$.
(3 pt)

2. Use Fermat's Theorem (or otherwise) to show that $42 | n^7 - n$ for all positive integers $n$.
(4 pt)

3. Compute $\phi(20!)$ (hint: first factor 20! in the form $20! = 2^a \cdot 3^b \cdot 5^c \cdots$).
(4 pt)

4. Let $n = 2537$. Suppose we also know that $\phi(n) = 2436$ and that $n = pq$ where $p, q$ are prime. Find $p$ and $q$ (without trying to factor $n$ by trial and error).
(4 pt)

5a) The letters of the alphabet were first encoded by: $A \rightarrow 1$, $B \rightarrow 2$, $C \rightarrow 3$, \ldots, $Z \rightarrow 26$. You would like to send an encoded message consisting of the letter “E” to a person who uses RSA with modulus $n = 437$ and exponent $e = 29$. What is the ciphertext $C$ that you send?
(3 pt)

5b) Using the fact that $437 = 23 \cdot 19$, compute the decoding exponent $d$ and use it to retrieve the plain text from the ciphertext $C$ obtained in a).
(4 pt)