Chapter 1: Preview and Review

1.1 Preliminaries

*NOTE this is not a complete list of material you are expected to know to be prepared for this course*

Real number line:

```
-2  -1  0  1  2
```

Interval notation

\[(a, b) = \{x : a < x < b\}\] open

and \([a, b] = \{x : a \leq x \leq b\}\] closed

The absolute value of a real number \(a\) is \(|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}\)

Example: Graph the function \(y = |4x - 8|\)

```
\[\begin{array}{c|c|c}
  x & y & 4(1) - 8 = 4 \\
  \hline
  -1 & \text{undefined} & \text{undefined} \\
  0 & -8 & -8 \\
  1 & 4(0) - 8 = 4 & 4 \\
  2 & 0 & 0 \\
  3 & 4 & 4
\end{array}\]
```
Chapter 1.1 Notes

A vertical line has no slope (undefined). All other lines have a slope given by the equation

\[ m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}} \]

**POINT-SLOPE form:** \[ y - y_1 = m(x - x_1) \]

**SLOPE-INTERCEPT form:** \[ y = mx + b, \ b \text{ is the } y\text{-intercept} \]

**GENERAL form:** \[ Ax + By + C = 0 \]

*Example:* A line has a slope of 2 and goes through the point (3, 4). What is the equation of the line? Graph the line and find the intercepts?

\[ y - y_1 = m(x - x_1) \]
\[ y - 4 = 2(x - 3) \]
\[ y = 2x - 6 + 4 \]
\[ y = 2x - 2 \]

The quantities \( x \) and \( y \) are proportional (\( y \propto x \)) if they are linearly related (\( y = mx \) or \( y - b = mx \)).
Chapter 1.1 Notes

Radian and Degrees: \(360^\circ = 2\pi\)

A few common angles

\[\begin{align*}
180^\circ &= \pi \\
90^\circ &= \frac{\pi}{2} \\
60^\circ &= \frac{\pi}{3}
\end{align*}\]

**Example:** Convert \(\frac{17\pi}{12}\) to degrees and convert \(50^\circ\) to radians

\[
\frac{17\pi}{12} \times \frac{360^\circ}{2\pi} = 255^\circ \\
50^\circ \times \frac{2\pi}{360^\circ} = \frac{5\pi}{18}
\]

Trigonometric Functions

\[
\begin{align*}
\sin \theta &= \frac{y}{r} \\
\cos \theta &= \frac{x}{r} \\
\tan \theta &= \frac{y}{x} \\
\csc \theta &= \frac{r}{y} \\
\sec \theta &= \frac{r}{x} \\
\cot \theta &= \frac{x}{y}
\end{align*}
\]

Pythagorean Theorem:

\[x^2 + y^2 = r^2\]
**Example:** (a) What is \( \cos \frac{\pi}{3} \)? (b) What is \( \sin \frac{\pi}{4} \)?

\[
\cos \frac{\pi}{3} = \frac{x}{r} = \frac{\text{ADJ}}{\text{HYP}} = \frac{1}{2} \\
\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}
\]

**Some Formulas and Identities**

\[
\tan \theta = \frac{\sin \theta}{\cos \theta} \\
\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}
\]

\[
\csc \theta = \frac{1}{\sin \theta} \\
\sec \theta = \frac{1}{\cos \theta}
\]

\[
\sin^2 \theta + \cos^2 \theta = 1 \\
\tan^2 \theta + 1 = \sec^2 \theta
\]

\[
\sin(-\theta) = -\sin \theta \\
\cos(-\theta) = \cos \theta
\]

\[
\sin(x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y
\]

\[
\cos(x + y) = \cos x \cdot \cos y + \sin x \cdot \sin y
\]
\[ f(x) = a^x, \quad a > 0, \quad a \neq 1 \] is called an exponential function

If \( a > 1 \), then this is exponential growth function where

- The domain is \(( -\infty, \infty )\)
- The range is \(( 0, \infty )\)

If \( 0 < a < 1 \), then this is exponential decay function where

- The domain is \(( -\infty, \infty )\)
- The range is \(( 0, \infty )\)

Properties to remember: Given that \( a \) and \( b \) are positive,

\[
a^{x+y} = a^x a^y \quad \quad (a^x)^y = a^{xy} \quad \quad (ab)^x = a^x b^x \quad \quad a^{-x} = \frac{1}{a^x}
\]
\[
\log_a x = y \iff a^y = x
\]

**Example:** Evaluate the following

(a) \(\log_2 64 = y \iff 2^y = 64 \Rightarrow y = 6\)

(b) \(\log_6 \left( \frac{1}{36} \right) = y \iff 6^y = \frac{1}{36} = \frac{1}{6^2} = 6^{-2} \Rightarrow y = -2\)

(c) \(\log_{16} 4 = y \iff 16^y = 4 \Rightarrow y = \frac{1}{2}\)

If \(x, y > 0\) then

\[
\log_a (xy) = \log_a x + \log_a y \quad \text{and} \quad \log_a (x^y) = y \log_a x
\]

Two special logarithms: \(\log_{10} x = \log x \quad \log_e x = \ln x = \ln x\)

Two useful formulas:

\[
\log_a (a^x) = x \quad a^{\log_e x} = x, \quad \text{for} \ x > 0 \quad \log_a x = \frac{\ln x}{\ln a}
\]
Example: Express the given quantity as a single logarithm

(a) \( \log_2 x + 5 \log_2 (x + 1) + \frac{1}{2} \log_2 (x - 1) \)

\[
= \log_2 x + \log_2 (x+1)^5 + \log_2 (x-1)^{\frac{1}{2}} \\
= \log_2 \left( x(x+1)^5(x-1)^{\frac{1}{2}} \right)
\]

(b) \( \ln x + a \ln y - b \ln z \)

\[
= \ln x + \ln y^a + \ln z^{-b} \\
= \ln \left( x y^a z^{-b} \right) \\
= \ln \left( \frac{x y^a}{z^b} \right)
\]

Example: Solve each equation for \( x \)

(a) \( \log (x+1) = 3 \) \( \Rightarrow \) \( \log_{10} (x+1) = 3 \) \( \Rightarrow \) \( 10^{\log_{10} (x+1)} = 10^3 \)

\( x+1 = 10^3 = 1000 \) \( \Rightarrow \) \( x = 999 \)

(b) \( e^{3x-4} = 2 \) \( \Rightarrow \) \( \ln e^{3x-4} = \ln 2 \) \( \Rightarrow \) \( 3x-4 = \ln 2 \)

\( \Rightarrow \) \( x = \frac{\ln 2 + 4}{3} \)

(c) \( \ln x + \ln (x-1) = 1 \)

\[
\ln \left( \frac{x}{x-1} \right) = 1 \\
\frac{\ln \left( \frac{x}{x-1} \right)}{e} = e^1 = e \\
\frac{x}{x-1} = e \Rightarrow x = e(x-1) - ex - e \\
1x - ex = -e \Rightarrow (1-e)x = -e \\
x = \frac{-e}{1-e} = \frac{e}{e-1}
\]

(d) \( \log x + \log (x+1) = \log 6 \)

\[
\log (x(x+1)) = \log 6 \\
x(x+1) = 6 \\
x^2 + x - 6 = 0 \\
(x-2)(x+3) = 0
\]

\( x = 2 \) or \( x = -3 \)

Not a solution

(c) Janice L. Epstein
A complex number is a number of the form
\[ z = a + bi \]
where \( a \) and \( b \) are real numbers and \( i^2 = -1 \). The real part is \( a \) and the imaginary part is \( b \).

Example: Find

(a) \( (7 - 2i) - (9 + 6i) = 7 - 2i - 9 - 6i = -2 - 8i \)

(b) \( (7 - 2i)(9 + 6i) = \frac{7(9) + 7(6i) - 2i(9) - 2i(6i)}{3 + 4i - 18i - 12i^2} \)
\[ = \frac{63 + 42i - 18i - 12}{12} \]
\[ = 75 + 24i \]

If \( z = a + bi \) is a complex number, then its conjugate is \( \bar{z} = a - bi \).

Example: Let \( z = 4 - 3i \). Compute

(a) \( \bar{z} = 4 + 3i \)

(b) \( \bar{z}z = (4 - 3i)(4 + 3i) = (4)(4) + (4)(3i) + (-3i)(4) + (-3i)(3i) \)
\[ = 16 + 9 = 25 \]

A quadratic equation is an equation in the form \( ax^2 + bx + c = 0 \). Where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \). The solution to this equation is
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Example: Solve \( 3x^2 + 2x = -5 \Rightarrow 3x^2 + 2x + 5 = 0 \)
\[ x = \frac{-2 \pm \sqrt{2^2 - 4(3)(5)}}{2(3)} = \frac{-2 \pm \sqrt{-56}}{6} = \frac{-2 \pm 2\sqrt{-14}}{6} \]
\[ = \frac{1}{3}(-1 \pm i\sqrt{14}) \]