**LOGIC**

A **proposition** or **statement** is a declarative sentence that can be classified as either true or false but not both.

**Examples**

- San Francisco is the capital of California.
- Be quiet!
- Texas is east of California or it is west of California.
- When is the next exam?
- The Exam 1 results were good.
- \(x + 1 = 4\)
- Miami Beach has \(10^{10}\) grains of sand.

A **prime** or **simple** proposition expresses one thought.

Join propositions with **logical connectives** to form compound propositions.

**conjunction**

**negation**

**disjunction**

\(p\): San Francisco is the capital of California.
\(q\): Austin is the capital of Texas.

What is \(p \land q\) in words? Is this proposition true or false?

In general,

Show this in a **truth table**,

\[
\begin{array}{ccc}
\text{p} & \text{q} & \text{p} \land \text{q} \\
\text{T} & \text{T} & \text{T} \\
\text{T} & \text{F} & \text{F} \\
\text{F} & \text{T} & \text{F} \\
\text{F} & \text{F} & \text{F} \\
\end{array}
\]

\(p\): San Francisco is the capital of California.
\(q\): Austin is the capital of Texas.

What is \(p \lor q\) in words? Is this proposition true or false?

In general,

\[
\begin{array}{ccc}
\text{p} & \text{q} & \text{p} \lor \text{q} \\
\text{T} & \text{T} & \text{T} \\
\text{T} & \text{F} & \text{T} \\
\text{F} & \text{T} & \text{T} \\
\text{F} & \text{F} & \text{F} \\
\end{array}
\]

This is called the **inclusive disjunction**. This is also the mathematical **or**.
Exclusive disjunction is $\lor$. This is true only if exactly one of the two statements is true.

$p$: San Francisco is the capitol of California.
$q$: Austin is the capitol of Texas.

What is $p \lor q$ in words? Is this proposition true or false?

$r$: Sacramento is the capitol of California.

What is $r \lor q$ in words? Is this proposition true or false?

Negation: $\neg p$ means not $p$. Show this in a truth table:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\neg p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Write the following statements symbolically and find the truth table.

The car is blue or has a moon roof.

$p$:
$q$:

The book is not red and the subject is history.

$p$:
$q$:

The sky is not blue or the grass is not purple.

$p$:
$q$: 

Show this in a truth table:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$r \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Define the following statements:

\( p \): The student is a girl.

\( q \): The student is a biology major.

\( r \): The student is enrolled in a math class.

Write the following statements symbolically and find the truth table.

The student is a boy and is not a biology major or enrolled in a math class.

A statement is a **contradiction** if the truth value of the statement is always false.

Example: Find the truth table for \( p \land \sim p \)

A statement is a **tautology** if the truth value of the statement is always true.

Example: Find the truth table for \( p \lor (\sim p \lor q) \)