CHAPTER 7: PROBABILITY

7.1: Experiments, Sample Spaces and Events

An experiment is an activity with an observable result. Tossing coins, rolling dice and choosing cards are all probability experiments.

The result of the experiment is called the outcome or sample point. The set of all outcomes or sample points is called the sample space of the experiment.

Example
What is the sample space for flipping a fair coin? Rolling a 6-sided die?

An event is a subset of a sample space. That is, an event can contain one or more outcomes that are in the sample space.

Example
What are all possible events for the experiment of flipping a fair coin?

Find the event E where $E = \{x | x \text{ has exactly one head}\}$

Find the event E where $E = \{x | x \text{ has two or more heads}\}$

Find the event E where $E = \{x | x \text{ has more than 3 heads}\}$

A sample space in which each of the outcomes has the same chance of occurring is called a uniform sample space.

Example
How many events are possible when a six-sided die is rolled?

Example
A bowl contains the letters AGGIES. How many outcomes are in the uniform sample space?
What is the uniform sample space for rolling two fair six-sided dice?

1~1   2~1   3~1   4~1   5~1   6~1
1~2   2~2   3~2   4~2   5~2   6~2
1~3   2~3   3~3   4~3   5~3   6~3
1~4   2~4   3~4   4~4   5~4   6~4
1~5   2~5   3~5   4~5   5~5   6~5
1~6   2~6   3~6   4~6   5~6   6~6

These sample spaces have all been finite. That is, we can list all the elements. An infinite sample space has to be described; you can't list all the elements:

Example
What is the sample space for the time spent working on a homework set?

Describe the event of spending between one and two hours on a homework set.

7.2 Definition of Probability

The probability of an event, \( P(E) \) is a number between 0 and 1, inclusive. If \( P(E) = 0 \), then the event \( E \) is impossible. If \( P(E) = 1 \), then the event \( E \) is certain.

The theoretical probability of an event \( E \) occurring is based on the sample space \( S \) having equally likely outcomes. Then probability of the event \( E \) occurring is

\[
P(E) = \frac{\text{number of outcomes in event } E}{\text{number of outcomes in the sample space }} = \frac{n(E)}{n(S)}
\]

Example
Consider flipping a fair coin three times. Find the following probabilities:

(a) Exactly one head is seen.

(b) Two or more heads are seen.

(c) More than 3 heads are seen.