On my honor, as an Aggie, I have neither given nor received unauthorized aid on this work.

Name (print):

In questions 1-6, show work!
In questions 7-8, write out the equations and ICs which you can then solve using Maple or by hand.

1. Cheburashka and Crocodile Gena are solving the differential equation

\[ x'' + x' - 6x = 4e^t. \]

Cheburashka’s general solution is

\[ x(t) = C_1e^{2t} + C_2e^{-3t} + e^t(e^t - 1), \]

while Gena’s is

\[ x(t) = C_1e^{-3t} + C_2e^{2t} + e^t(e^{-4t} - 1). \]

Who is right? Why? (4 points)
2. For the equation
\[ \frac{dx}{dt} = 1 - x^2. \]

(a) Find the general solution (explicit!).

(b) Find the particular solutions satisfying \( x(0) = 0 \), \( x(0) = -1 \) and \( x(0) = 1 \).

(6 points)
3. Solve the IVP (implicit solution is ok)

\[ y \, dx + (x + 3y^2) \, dy = 0, \quad y(0) = 1. \]

(6 points)
4. Solve the IVP (implicit solution is ok)

\[ 2x^2 \, dy - (y + x)^2 \, dx = 0, \quad y(1) = 1. \]

(6 points)
5. Solve the IVP

\[(x + 1) \frac{dy}{dx} - y = (x + 1)^2, \quad y(0) = -1.\] (6 points)
6. Using a substitution, solve the IVP (explicit solution!)

\[
\frac{dy}{dx} = y - x - 1 + \frac{1}{x - y + 2}, \quad y(0) = 4.
\]

(6 points)
7. Pure water is being pumped into tank X at the rate 2 L/min. The solution in tank X is kept well stirred and flows out into tank Y at the same rate, 2 L/min. In tank Y it is stirred again and flows out at the same rate. If initially tank X contained 20 L of brine at the concentration 0.1 kg/L and tank Y contained 40 L of pure water, what is the maximum concentration ever reached in tank Y?

(8 points)
8. A rocket weighing 20kg is being launched vertically from the ground. It has 10kg of fuel (included in the launch mass of 20kg) which is being used by the rocket engine at the rate 1kg/sec. The engine creates thrust of $50g$ Newtons, where $g$ is the gravitational constant. Ignoring the air resistance, find the velocity of the rocket when the fuel runs out. (Hint: Take care when using Newton’s second law $\frac{d}{dt}(mv) = F$).

(8 points)