Homework 9: Sections 5.1-5.3

Show ALL work on your own paper to get full credit. This assignment is due at the beginning of class on **Wednesday, April 13**.

1. (5.1) #12 from Section 5.1 in the book. Assume domain is \( \mathbb{R} \). (Don’t turn in graph for part (e).)

2. (5.1) Sketch a graph of a function \( f \) that satisfies the following conditions. Feel free to use the graph provided on page 2.
   - \( f(0) = 3, f(5) = 4, f(7) = 6 \)
   - \( \lim_{x \to \pm \infty} f(x) = 0 \)
   - \( \lim_{x \to 3^{-}} f(x) = -\infty, \lim_{x \to 3^{+}} f(x) = \infty \)
   - \( f'(x) > 0 \) on the intervals \( (-3, 0) \cup (5, 7) \).
   - \( f'(x) < 0 \) on the intervals \( (-\infty, -3) \cup (0, 3) \cup (3, 5) \cup (7, \infty) \)
   - \( f''(x) > 0 \) on the intervals \( (-\infty, -5) \cup (3, 7) \cup (7, \infty) \)
   - \( f''(x) < 0 \) on the intervals \( (-\infty, -1) \cup (-1, 3) \)

3. (5.2) Consider the piecewise function \( f(x) = \begin{cases} -x-2 & \text{if } -2 \leq x < 0 \\ \frac{x}{2} & \text{if } 0 \leq x < 2 \\ (x-2)^2 & \text{if } 2 \leq x \leq 4 \end{cases} \)
   - Sketch a graph of the function. Feel free to use the graph provided on page 2.
   - Identify the absolute maximum and minimum values if they exist and state where they occur.
   - Identify the \( x \)-coordinates where the function has a local maximum or local minimum.

4. (5.2) Find the critical numbers of the function \( f(x) = \sqrt{x(x-1)^4} \).
   (Hint: After differentiating, factor and then get a common denominator.)

5. (5.2) Find the absolute maximum and minimum values of the given function \( f \) on the indicated closed interval.
   - (a) \( f(x) = \sqrt{9 - x^2} \) on the interval \([-1, 2]\)
   - (b) \( f(x) = xe^{-x} \) on the interval \([0, 2]\)

6. (5.3) Given the function \( f(x) = \ln(x+1) \) on the interval \([0, 2]\), show that \( f \) satisfies the Mean Value Theorem.

7. (5.3) Given the function \( f(x) = e^{x^2-5x^3} \), find the intervals where \( f \) is increasing and decreasing and identify the \( x \)-values where the function has a local max or min, if any.

8. (5.3) Given the function \( f(x) = \ln(x^2 + 1) \), find the intervals where \( f \) is concave up and concave down and identify the \( x \)-values where the function has an inflection point, if any.

9. (5.3) For the following two functions, find the indicated information and sketch a graph.
   - (a) \( f(x) = x^5 - 2x^4 \)
   - (b) \( f(x) = \frac{x^2}{x^2 - 4} \). I’ll let you find \( f'(x) \), but \( f''(x) = \frac{8(3x^2 + 4)}{(x-2)^3(x+2)^3} \)
     i. Domain
     ii. Vertical and horizontal asymptotes
     iii. \( x \) and \( y \)-intercepts
     iv. Intervals where \( f \) is increasing/decreasing
     v. \( x \)-values where \( f \) has a local max/min
     vi. Intervals where \( f \) is concave up/down
     vii. \( x \)-values where \( f \) has an inflection point
For #2:

For #3a:

For #9a:

For #9b: