Homework 7: Sections 3.10, 3.11, & 4.1

Show ALL work on your own paper to get full credit. This assignment is due at the beginning of class on **Wednesday, March 23**.

1. (3.10) A hot air balloon takes off vertically and maintains a constant speed of 10 mph. A man is watching the take-off from a point 2 miles away.

   (a) At what rate is the angle of elevation from the man to the balloon changing when that angle is $\frac{\pi}{6}$?

   (b) At what rate is the distance from the man to the balloon changing after 15 minutes?

2. (3.10) Water is being poured into an inverted conical tank at a constant rate of 10 m$^3$/min. Water is also leaking out of the tank at some unknown constant rate. The tank has a height of 40 m and a radius of 5 m. If it is known that the water level is decreasing at a rate of 2 m/min when the height of the water is 15 m, find the rate at which the water is leaking out of the tank.

3. (3.10) A trough is 10 ft long and its ends have the shape of isosceles triangles that are 6 ft across the top and have a height of 4 ft. If the trough is filled with water at a rate of 12 ft$^3$/min, how fast is the water level rising when the width of water across the top of the trough is 2 ft?

4. (3.10) The right triangle below has the property that $x$ is decreasing at a rate of 2 in/sec.

   \[ \begin{array}{ccc}
   & & z \\
   & x & \\
   y & & \\
   \end{array} \]

   (a) Suppose the area of the triangle remains a constant 24 in$^2$. At what rate must $y$ be increasing when $x = 4$? (You don’t need to use $z$ in part (a).)

   (b) At what rate is the hypotenuse, $z$, changing at this same moment?

5. (3.11) Consider the function $f(x) = 1 + x^2$. Find $\Delta y$ and $dy$ when $x = 4$ and $\Delta x = 0.25$.

   What is the error in using differentials to approximate $f(4.25)$?

6. (3.11) Use differentials to find an approximate value for $\sqrt{26.9}$. Express your answer as an exact fraction.

7. (3.11) Find the linearization of $f(x) = \frac{1}{(x + 1)^2}$ at $a = 2$ and use it to approximate $\frac{1}{3.01^2}$. Express your answer as an exact fraction.

8. (3.11) Find the quadratic approximation of $f(x) = \cos x$ at $a = \frac{\pi}{3}$.

9. (4.1) Calculate the following limits.

   \[ \begin{array}{ccc}
   (a) \lim_{x \to -\infty} \pi^{-x} & (b) \lim_{x \to -2} \left( \frac{5}{2} \right)^{x+3} & (c) \lim_{x \to \infty} \frac{2e^{3x} - e^{-4x}}{3e^{3x} + 5e^{-4x}} \\
   & & (d) \lim_{x \to -\infty} \frac{2e^{3x} - e^{-4x}}{3e^{3x} + 5e^{-4x}} \\
   \end{array} \]

10. (4.1) Differentiate the following. Do Not Simplify.

    (a) $f(x) = e^{\sqrt{\sin x}} \cos(3e^x)$

    (b) $g(x) = \frac{e^{-5x}}{e + \tan(e^{x^2-x})}$

11. (4.1) Find $f^{(n)}(x)$ for $f(x) = -e^{-3x}$.

12. (4.1) Find the slope of the tangent line to the curve $e^{x^2+y} + \cos(e^y - 1) = 2x^2 - y$ at the point $(1, 0)$. 