Math 166 Exam 3 Review
Sections F.1-F.4, 4.3-4.4, & 5.1-5.3
Note: Some instructors did not get to 5.3

Note: This review covers the highlights of these sections, not every type of problem that could be on an exam. For additional practice problems, take a look at Week in Reviews 7 and 8.

1. Solve for the variables in the following equation.
\[
\begin{bmatrix}
1 & -2 \\
-4 & 9
\end{bmatrix}
- 2 \begin{bmatrix}
c + 1 & 6 \\
2b & a
\end{bmatrix}^T
= \begin{bmatrix}
-1 & -5 \\
9d & 4
\end{bmatrix}
\begin{bmatrix}
3 & 0 \\
1 & 4
\end{bmatrix}
\]

2. Given the following matrices and their sizes, determine whether the matrix operations are possible or not possible. If possible, give the dimensions (order) of the resulting matrix.

\[A_{2\times4} \quad B_{3\times2} \quad C_{4\times3} \quad D_{3\times3} \quad E_{2\times3} \quad F_{2\times2}\]

(a) \(4AC + E\)  
(b) \(BA + C\)  
(c) \(A^T E - 7C\)  
(d) \(DB^T\)

True or False:

(e) \(I_2 F = F\)  
(f) \(BI_3 = B\)
(From 5.3) Suppose you are given the additional information that $D$ is nonsingular and $F$ is singular:

$$A_{2\times 4} \quad B_{3\times 2} \quad C_{4\times 3} \quad D_{3\times 3}, \text{nonsingular} \quad E_{2\times 3} \quad F_{2\times 2}, \text{singular}$$

Determine whether the matrix operations are possible or not possible. If possible, give the dimensions (order) of the resulting matrix.

(g) $BED^{-1}$  
(h) $F^{-1}A$  
(i) $A^{-1}F$  
(j) True or False: $DD^{-1} = 1$

3. John and Bill are both throwing Super Bowl parties and want to order pizzas. Matrix $A$ gives the cost of a small, medium, and large pizza at Papa John’s and Pizza Hut. Matrix $B$ gives the number of small, medium, and large pizzas that John and Bill want to order for their parties.

$$A = \begin{bmatrix} PJ & PH \\ S & 6 \\ M & 9 \\ L & 12 \end{bmatrix} \quad B = \begin{bmatrix} J & B \\ S & 5 \\ M & 3 \\ L & 6 \end{bmatrix}$$

Which of the following matrix products gives the cost of each person’s order if they ordered at each store?

• $AB$  
• $BA$  
• $AB^T$  
• $B^TA$
4. Matrix $X$ represents the number of cups of sugar and flour per serving of food I and food II. Matrix $Y$ gives the number of servings of food I and food II needed for event A and event B.

\[
X = \begin{bmatrix}
3 & 7 \\
2 & 9 \\
\end{bmatrix}
\quad Y = \begin{bmatrix}
12 & 10 \\
6 & 11 \\
\end{bmatrix}
\]

(a) What is the meaning of the matrix $XY$?

(b) What is the meaning of the matrix $YX$?

5. Solve the following matrix equation for $X$. (From 5.3)
(assume all matrix operations are possible and that any necessary inverses exist.)

\[
X - XB + E = F
\]

6. Solve the following systems of equations. If there are infinitely many solutions, give a parameterized solution and a particular solution.

(a) $10x - 8y = 6$
   $-15x + 12y = 20$
   $-25x + 20y = 72$
(b) \[ x + y + z + u = 5 \]
\[ 2x + 3y - 9z = 18 \]
\[ 3x + 16z + 4u = 1 \]

7. Suppose there are 3 major airlines that fly out of Houston: Southwest, Delta, and American. A Southwest flight holds 135 people, a Delta flight holds 160 people, and an American flight holds 148 people. On a given day, a total of 88,683 people flew out of Houston. There were 3 times as many Southwest flights as Delta flights, and the number of American flights was half the number of Southwest and Delta flights combined. How many flights out of Houston were there for each airline if every flight was full?

Solve your system by creating a matrix equation \( AX = B \) and using matrix inverses. (From 5.3) If you have not covered 5.3, use an augmented matrix and rref.
8. An event center sold student, adult, and senior citizen tickets for a concert. A student ticket cost $40, an adult ticket cost $85, and a senior citizen ticket cost $70. They sold two times as many student tickets as senior citizen tickets, the amount of money made from adult tickets was 8.5 times the amount of money made from student tickets, and they brought in total revenue of $872,330. How many tickets of each type were sold?

9. A club is making three different candy bags to give away for Halloween. Each Bag A will have 1 Tootsie Roll, 2 Snickers, and 4 Twix. Each Bag B will have 3 Tootsie Rolls, 5 Snickers, and 6 Twix. Each Bag C will have 5 Tootsie Rolls, 7 Snickers, and 2 Twix. They have available 82 Tootsie Rolls, 134 Snickers, and 148 Twix. If they use all their supply, how many of each type of bag can they make?
10. Farmer Fran invested some money at a simple interest rate of 4%/yr. After 21 months, the account has $700 in it. How much was initially invested?

11. Farmer Fran has $10,000 to invest. He chooses to invest this money in an account with an interest rate of 2%/yr compounded quarterly.
   
   (a) How much interest will he have earned after 7 years?
   
   \[
   \begin{align*}
   N &= \quad \text{PMT}= \\
   I\% &= \quad \text{FV}= \\
   PV &= \quad \text{P/Y}=C/Y= 
   \end{align*}
   \]

   (b) How many years until his investment doubles?
   
   \[
   \begin{align*}
   N &= \quad \text{PMT}= \\
   I\% &= \quad \text{FV}= \\
   PV &= \quad \text{P/Y}=C/Y= 
   \end{align*}
   \]

   (c) If the account was compounded continuously instead, how much would he have after the above amount of time?

12. Farmer Fran wants to save up some money to buy a new tractor. He knows he’ll need $15,000. He plans to invest some money each month in an account that earns 3.1%/yr compounded monthly so that he has the $15,000 in 2 years.
   
   (a) How much should he put in the account each month?
   
   \[
   \begin{align*}
   N &= \quad \text{PMT}= \\
   I\% &= \quad \text{FV}= \\
   PV &= \quad \text{P/Y}=C/Y= 
   \end{align*}
   \]
(b) How much total interest is earned in the 2 years?

(c) How much interest is earned during the 3rd month of the second year?

13. When you retire, you will receive $1000 weekly payments for 20 years from an account that earns interest at a rate of 4.3%/yr compounded weekly?

(a) How much needs to be in the account when you retire?
   \[ \begin{align*}
   N &= \quad \text{PMT}= \\
   I\% &= \quad \text{FV}= \\
   PV &= \quad P/Y=C/Y= \\
   \end{align*} \]

(b) What is the effective interest rate for this account?

14. Farmer Fran put down $10,000 on a $190,000 house and financed the remaining balance with a 15-year mortgage at an interest rate of 5.4%/yr compounded monthly.

(a) What is his monthly payment?
   \[ \begin{align*}
   N &= \quad \text{PMT}= \\
   I\% &= \quad \text{FV}= \\
   PV &= \quad P/Y=C/Y= \\
   \end{align*} \]

(b) How much will he pay in interest over the 15 years?
(c) What is the outstanding principal after 10 years? What is his equity?

\[
\begin{align*}
N &= \quad \text{PMT} = \\
I\% &= \quad \text{FV} = \\
PV &= \quad \text{P/Y}=C/Y = \\
\end{align*}
\]

(d) Fill in the partial amortization table for the first two periods below.

<table>
<thead>
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<th>Period</th>
<th>Payment</th>
<th>Towards Interest</th>
<th>Towards Principal</th>
<th>Outstanding Principal</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
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<td>1</td>
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</tr>
<tr>
<td>2</td>
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</tbody>
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