Math 166 Week in Review 8
Sections F.4b, 4.3, & 4.4

1. Ben decides to buy a house. The house costs $240,000. He makes a 10% down payment and takes out a mortgage on the remaining balance. The mortgage loan is for 30 years at an interest rate of 6.75%/yr compounded monthly.

(a) How much will the monthly mortgage payment be?

\[
\text{Down payment} = 0.1 \times (240,000) = 24,000
\]

\[
\text{Loan} = 240,000 - 24,000 = 216,000
\]

\[
\text{Present Value of Loan} = 216,000
\]

\[
\text{Interest} = 504349.20 - 216000 = 288349.20
\]

(b) How much would he pay in interest over the life of the loan?

\[
\text{Paid on loan out of pocket: } (1400.97)(360) = 504,349.20
\]

\[
\text{Value of loan: } 216,000
\]

(c) What is the outstanding principal after 13 years? What is his equity at this point?

\[
\text{Let } N = \text{ # of remaining payments.}
\]

\[
\text{Equity: } \text{Value of Item} - \text{OP} = 240,000 - 169747.35 = 70,252.65
\]

After these 13 years, Ben decides to refinance his home. The new loan on the remaining balance is to be paid off over 20 years and has an interest rate of 5.6%/yr compounded monthly.

(d) What would be the new monthly mortgage payment?

\[
\text{New Loan: } (1177.28)(20)(12) = 282,547.20
\]

(e) How much money will he save by refinancing?

\[
\text{Old Loan: } (1400.97)(17)(12) = 285,797.88
\]

\[
\text{New Loan: } 3250.68
\]
2. Ben makes a foolish purchase. He buys a high definition flat screen TV using a credit card. The TV cost $2600. The credit card has an interest rate of 18%/yr compounded monthly. After 1 month, Ben gets his first credit card bill. The statement suggests that he make the minimum payment of $94.

(a) If Ben makes the first payment of $94, how much of this goes toward interest?

\[ \text{monthly interest rate} \times 2600 \left( \frac{0.18}{12} \right) = \text{ } 39 \]

(b) How much of the first payment goes towards principal? What does he owe on the card after the 1st payment?

\[ \$94 - \$39 = \$55 \rightarrow \text{to principal} \]

Now owe: \[ 2600 - 55 = \$2545 \]

(c) How much of the second payment goes towards principal?

\[ \text{Interest: } (2545) \left( \frac{0.18}{12} \right) = 38.18 \]

To principal: \[ 94 - 38.18 = \$55.82 \]

(d) If he makes $94 payments each month, how long will it take him to pay the TV off?

\[ N = \frac{PMT}{\text{P/Y}} \times FV \]

\[ N = \frac{94}{12} \times 0 = \frac{36}{3} = 36 \text{ months} = 3 \text{ years} \]

(e) How much total interest will he end up paying?

Out of pocket: \[ (94)(36) = 3384 \leftarrow \]

Interest: \[ 3384 - 2600 = \$784 \]

(f) If Ben decides he wants to pay off the TV in 16 months, how much should his monthly payment be?

\[ N = \frac{16}{12} \times P/Y \times FV \]

\[ N = \frac{16}{12} \times 0 = \frac{16}{12} \times \text{P/Y} = \text{PMT} \]

\[ \text{PMT} = \$183.99 \]

(g) How much interest does he save by paying it off in 16 months than by making $94 payments?

Under $94 payments, he will pay \[ 3384 \]

Under new payments: \[ (183.99)(16) = 2943.84 \]

\[ 3384 - 2943.84 = \$440.16 \]
3. Ben buys a $25,000 car. He makes a down payment of $5000 and takes out a loan on the balance. He is required to make semiannual payments for 10 years at an interest rate of 3.2%/yr compounded semiannually. Fill in the amortization table for the first 2 years (4 periods) of the loan.

\[
\begin{align*}
N &= 20 \\
I &= 3.2 \% \\
P &= 20000 \\
P/M &= 0 \\
P/Y = C/Y &= 2
\end{align*}
\]

<table>
<thead>
<tr>
<th>Period</th>
<th>Payment</th>
<th>Towards Interest</th>
<th>Towards Principal</th>
<th>Outstanding Principal</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>20,000</td>
<td>5,000</td>
</tr>
<tr>
<td>1</td>
<td>1176.43</td>
<td>320</td>
<td>856.43</td>
<td>19143.57</td>
<td>5856.43</td>
</tr>
<tr>
<td>2</td>
<td>1176.43</td>
<td>306.30</td>
<td>870.13</td>
<td>18273.44</td>
<td>6726.56</td>
</tr>
<tr>
<td>3</td>
<td>1176.43</td>
<td>292.38</td>
<td>884.05</td>
<td>17389.39</td>
<td>7610.61</td>
</tr>
<tr>
<td>4</td>
<td>1176.43</td>
<td>278.23</td>
<td>898.20</td>
<td>16491.19</td>
<td>8508.81</td>
</tr>
</tbody>
</table>

**Payment 1:**

Interest: \((20,000) \left( \frac{0.032}{2} \right) = 320\)

Principal: \(1176.43 - 320 = 856.43\)

\(OP = 20,000 - 856.43 = 19143.57\)

Equity: \(5000 + 856.43 = 5856.43\)

**Payment 2:**

Interest: \((19143.57) \left( \frac{0.032}{2} \right) = 306.30\)

Principal: \(1176.43 - 306.30 = 870.13\)

What is his equity after 8 years?

\[
\begin{align*}
N &= 2.2 \\
P/M &= -1176.43 \\
I &= 3.2 \% \\
P &= 0 \\
P/Y = C/Y &= 2
\end{align*}
\]

\(OP = 4523.35\)

Equity = Value of item - OP

\(25000 - 4523.35 = 20476.65\)
4. Solve the following systems of equations. If there are infinitely many solutions, provide a parametric form of the solution.

(a) \( x = -y - 2 \)
\( x + y + z = -2 \)
\( 3x + 2y + 2z = -3 \)
\( z = 2 - 2x \)

\[
\begin{bmatrix}
1 & 1 & 0 & -2 \\
1 & 3 & 2 & -3 \\
2 & 0 & 1 & 2
\end{bmatrix}
\xrightarrow{\text{ref}}
\begin{bmatrix}
1 & 0 & 1 & -3 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]
\( x = 1 \)
\( y = 3 \)
\( z = 0 \)
\( (x, y, z) = (1, -3, 0) \)

(b) \( x + 2y + z = 3 \)
\( 3x + 3y + 3z = 7 \)
\( 2x + y + 2z = 1 \)

\[
\begin{bmatrix}
1 & 2 & 1 & 3 \\
3 & 3 & 1 & 7 \\
2 & 1 & 2 & 1
\end{bmatrix}
\xrightarrow{\text{ref}}
\begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1
\end{bmatrix}
\]
\( x + z = 0 \)
\( y = 0 \)
\( z = 1 \)
\( \text{NO SOLUTION} \)

(c) \( 2x + y - z = 0 \)
\( x - 3y + z = 1 \)
\( x + 4y - 2z = -1 \)

\[
\begin{cases}
\frac{x}{2} - \frac{z}{2} = \frac{1}{7} \\
y - \frac{3z}{2} = -\frac{2}{7}
\end{cases}
\]
Let \( z = t \)
\[
\begin{align*}
\frac{x}{2} - \frac{z}{2} &= \frac{1}{7} \\
y - \frac{3z}{2} &= -\frac{2}{7}
\end{align*}
\]
\[
\begin{align*}
x &= \frac{1}{7} + \frac{2}{7}t \\
y &= \frac{2}{7} + \frac{3}{7}t \\
z &= t
\end{align*}
\]
\( (x, y, z) = \left( \frac{1}{7} + \frac{2}{7}t, -\frac{2}{7} + \frac{3}{7}t, t \right) \), where \( t \) is any real #.

(d) \( x + y + 2z - 3u = 4 \)
\( -2x + y + z + u = 2 \)
\( -x + 2y + 3z - 2u = 6 \)

\[
\begin{bmatrix}
1 & 0 & 1 & 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\
0 & 1 & 1 & 1 & \frac{5}{3} & -\frac{5}{3} & -\frac{5}{3} \\
0 & 0 & 1 & 0 & \frac{2}{3} & -\frac{1}{3} & \frac{4}{3}
\end{bmatrix}
\xrightarrow{\text{ref}}
\begin{bmatrix}
1 & 0 & 1 & 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\
0 & 1 & 1 & 1 & \frac{5}{3} & -\frac{5}{3} & -\frac{5}{3} \\
0 & 0 & 1 & 0 & \frac{2}{3} & -\frac{1}{3} & \frac{4}{3}
\end{bmatrix}
\]
\[
\begin{align*}
x + \frac{1}{3}z &= \frac{2}{3} \\
y + \frac{5}{3}z - \frac{5}{3}u &= \frac{10}{3}
\end{align*}
\]
Let \( z = t \), \( u = s \)
\[
\begin{align*}
x &= \frac{2}{3} - \frac{1}{3}t + \frac{4}{3}s \\
y &= \frac{10}{3} - \frac{5}{3}t + \frac{5}{3}s
\end{align*}
\]
\( (x, y, z, u) = \left( \frac{2}{3} - \frac{1}{3}t + \frac{4}{3}s, \frac{10}{3} - \frac{5}{3}t + \frac{5}{3}s, t, s \right) \)
where \( t \) and \( s \) are any real #s.
5. The following reduced augmented matrices represent systems with infinitely many solutions. Give a parametric form of the solution and list 2 particular (specific) solutions. (Assume the columns represent the variables x, y, z, and u if necessary.)

(a) \[
\begin{bmatrix}
1 & 2 & 0 & 4 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
\[
x + 2y = 4, \quad z = 2 \quad \text{Let } y = t \\
x + 2t = 4 \\
x = 4 - 2t \\
(x, y, z) = (4 - 2t, t, 2) \text{ where } t \text{ is any real number.}
\]
\[
t = 0: (4, 0, 2) \\
t = 1: (2, 1, 2) \\
\]

(b) \[
\begin{bmatrix}
0 & 1 & 0 & 3 & 4 \\
0 & 0 & 1 & 2 & 5 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
\[
y + 3u = 4 \\
\quad \text{Let } x = t, \\
y + 3s = 4 \\
z + 2u = 5 \\
y = 4 - 3s \\
z = 5 - 2s \\
(x, y, z, u) = (t, 4 - 3s, 5 - 2s, s) \text{ where } t + s \text{ are any real numbers.}
\]
\[
t = 0, s = 0: (0, 4, 5, 0) \\
t = 1, s = 2: (1, -2, 1, 2)
\]
6. A system of equations has infinitely many solutions parameterized by \((x, y, z, w) = (9 + 2t, t, t - 3s, s)\). Determine whether the following are particular solutions to this system.

(a) \((11, 1, -5, 2)\)

\[
\begin{align*}
q + 2t &= 9 + 2(11) = 29 \checkmark \\
t - 3s &= 1 - 3(2) = -5 \checkmark
\end{align*}
\]

Yes

(b) \((7, -1, 4, 1)\)

\[
\begin{align*}
t &= -1, \quad s = 1 \\
q + 2t &= 9 + 2(-1) = 7 \checkmark \\
t - 3s &= -1 - 3(1) = -4 \times
\end{align*}
\]

NO
7. Suppose I had a drink stand where I sold bottles of lemonade, Koolaid, and Gatorade. I sold each bottle of lemonade for $1.25, each bottle of Koolaid for $1.75, and each bottle of Gatorade for $2.50. At the end of a certain day, I had sales of $254. In all I had sold 158 bottles, and I knew that I sold three times as many lemonade bottles as Gatorade bottles. How many bottles of each drink did I sell?

\[
\begin{align*}
\text{Let } & x = \text{ # of bottles of lemonade sold} \\
& y = \text{ # of bottles of Koolaid sold} \\
& z = \text{ # of bottles of Gatorade sold}
\end{align*}
\]

\[
1.25x + 1.75y + 2.50z = 254
\]

\[
x + y + z = 158
\]

\[
\begin{bmatrix}
1.25 & 1.75 & 2.50 \\
1 & 1 & 1 \\
1 & 0 & -3
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
90 \\
38 \\
30
\end{bmatrix}
\]

\[
(x, y, z) = (90, 38, 30)
\]
8. Ben has a bunch of five, ten, and twenty dollar bills in his pocket. There are one third as many 10s and 20s combined as there are 5s. Additionally, the amount of money in 5s is $70 more than the amount of money in 10s, and the amount of money in 20s is 4 times the amount of money in 10s. How many of each bill does he have? Just set up the system.

Let $x =$ number of 5-dollar bills, $y =$ number of 10-dollar bills, and $z =$ number of 20-dollar bills.

\[
y + z = \frac{1}{3}x
\]

\[
5x = 70 + 10y
\]

\[
20z = 4(10y)
\]
9. Freebirds offers regular, monster, and super monster burritos. The number of servings of rice, beans, and meat in each type of burrito is given below in the table. If Freebirds has 1085 servings of rice, 1410 servings of beans, and 1195 servings of meat available on a given day, how many of each size burrito can be made if all the food is used? Just set up the system.

Let \( x = \) number of regulars, \( y = \) the number of monsters, and \( z = \) number of super monsters made.

<table>
<thead>
<tr>
<th></th>
<th>Rice</th>
<th>Beans</th>
<th>Meat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Monster</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Super Monster</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Available: 1085, 1410, 1195

\[
\begin{align*}
x + 2y + 3z &= 1085 \\
2x + 2y + 3z &= 1410 \\
x + 2y + 4z &= 1195
\end{align*}
\]
10. Ben invested some money in three different accounts that have 3%, 5%, and 8% annual effective yields. At the end of the year, he had earned a total of $228 in interest. There was twice as much interest earned from the 8% account as from the 3% and 5% accounts combined and the amount of money invested in the 3% account was 1.5 times the amount of money invested in the 5% account. How much was invested in each account? Define variables and set up the system.

Let $x = \text{ amount invested at } 3\%$
$y = \text{ amount invested at } 5\%$
$z = \text{ amount invested at } 8\%$

\[
.03x + .05y + .08z = 228
\]
\[
.08z = 2(.03x + .05y)
\]
\[
x = 1.5y
\]
11. A pizza store makes small, medium, and large pizzas. A small pizza takes 4 minutes baking time, a medium pizza takes 12 minutes baking time, and a large pizza takes 28 minutes baking time. Every hour, the store has 4 hours of baking time available due to multiple ovens. Additionally, the store knows they must make 12 pizzas each hour to meet customer demands. How many pizzas of each size can the store make each hour if all baking time is used?

Let $x =$ number of small pizzas, $y =$ number of medium pizzas, $z =$ number of large pizzas

\[
\begin{align*}
4x + 12y + 28z &= 240 \\
x + y + z &= 12
\end{align*}
\]

\[
\begin{bmatrix}
4 & 12 & 28 & 240 \\
1 & 1 & 1 & 12
\end{bmatrix}
\xrightarrow{\text{rref}}
\begin{bmatrix}
1 & 0 & -2 & -12 \\
0 & 1 & 3 & 24
\end{bmatrix}
\]

$x - 2z = -12$

$y + 3z = 24$

Let $z = t$, $x = -12 + 2t$, $y = 24 - 3t$

$(x, y, z) = (-12 + 2t, 24 - 3t, t)$, where $t = 6, 7, 8$

$t = 0: \left(8, 24, 0\right)$

$t = 6: \left(0, 6, 6\right)$

$t = 7: \left(2, 3, 7\right)$

$t = 8: \left(4, 0, 8\right)$