Math 166 Week in Review 7
Sections F.1-F.4a

1. When Ben is in 2nd grade, his parents open up a bank account for him that earns interest at a simple interest rate of 8.5% per year. Ben puts $20 in the account to begin with. His goal is to have $71 in his account so he can buy a Gameboy. How many years will it take to have $71 in the account?

\[ r = \frac{.085}{1} \]
\[ P = 20 \]
\[ F = 71 \]
\[ t = ? \]

\[ I = Prt \]
\[ F = P(1+rt) \]
\[ I = F - P = 51 \]
\[ I = Prt \]
\[ 51 = 20(.085)t \]
\[ t = 30 \text{ yrs} \]
2. Ben doesn’t have this kind of time, so he decides to open up a different bank account with a better simple interest rate of 11%. If Ben wants to have $71 in 15 months, how much should he invest in this new account?

\[ F = P(1 + rt) \]

\[ 71 = P(1 + 0.11 \cdot \frac{15}{12}) \]

\[ 71 = 1.1375P \]

\[ P = \frac{71}{1.1375} = 62.42 \]

3. Ben doesn’t have this kind of money, so he changes accounts again. This time he starts his account with $30 (his parents helped out) and he plans on just leaving it there for a while. In 50 months, Ben has $39.25 in his account. What was the simple interest rate on this account?

\[ F = P(1 + rt) \]

\[ 39.25 = 30(1 + rt) \]

\[ I = F - P = 9.25 \]

\[ I = Prt \]

\[ 9.25 = 30 \cdot r \cdot \frac{50}{12} \]

\[ r = 0.074 \]

\[ 7.4\% \]
4. Ben is now in 7th grade and is becoming smarter. He decides that simple interest rates aren’t good. So he takes his $209 birthday money and puts it in a savings account that earns interest at a rate of 9%/year compounded quarterly.

(a) How much will Ben have at the end of 3 years?

\[ N = 9 \times 4 \quad PMT = 0 \]
\[ I\% = 9 \quad FV = ? \]
\[ PV = 209 \quad P/Y = C/Y = 4 \]
\[ FV = \$272.96 \]

(b) How much interest is earned in total?

\[ I = 272.96 - 209 = \$63.96 \]

(c) How much interest is earned in the 3rd quarter of the 3rd year?

\[ N = 10 \quad PMT = 0 \]
\[ I\% = 9 \quad FV = ? \]
\[ PV = 209 \quad P/Y = C/Y = 4 \]
\[ FV = 266.96 \]
\[ \text{1st year: } 1 \text{ - 4} \]
\[ \text{2nd year: } 5 \text{ - 8} \]
\[ \text{3rd year: } 9 \text{ - 10} \]
\[ \text{Interest} = 266.96 - 261.08 = \$5.88 \]

(d) What would the interest rate have to be (rounded to 4 decimal places) if he wants to have $300 at the end of the 3 years and interest is still compounded quarterly?

\[ N = 12 \quad PMT = 0 \]
\[ I\% = ? \quad FV = 300 \]
\[ PV = -209 \quad P/Y = C/Y = 4 \]
\[ I = 12.231620 \]
5. After 10th grade, Ben gets a part time job. At the end of each month he deposits $125 from his paycheck into a savings account that has an interest rate of 8.65% compounded monthly.

(a) How much is in the account after 2 years?

\[
\begin{align*}
N &= 24 \\
I% &= 8.65 \\
\text{PMT} &= -125 \\
\text{PV} &= 0 \\
\text{FV} &= ? \\
\text{P/Y} = \text{C/Y} &= 12 \\
\text{FV} &= 3262.35
\end{align*}
\]

(b) How much total interest was earned?

What was put in? \((125)(24) = 3000\)

\[
\text{Interest} = 3262.35 - 3000 = 262.35
\]

(c) How much interest was earned in the 3rd month of the second year?

\[
\begin{align*}
\text{FV} &= 1972.63 \\
\text{1st year: } 1 - 12 \\
\text{2nd year: } 13 - 14 - 15 \\
\text{FV} &= 1834.41
\end{align*}
\]

During 15th month, account increased by $138.22, but $125 of this was deposited. So interest = $138.22 - $125 = $13.22

(d) In how many months will Ben have $3500 in this account?

\[
\begin{align*}
\text{FV} &= 3500 \\
\text{N} &= ? \\
\text{I%} &= 8.65 \\
\text{PMT} &= -125 \\
\text{PV} &= 0 \\
\text{P/Y} = \text{C/Y} &= 12 \\
\text{N} &= 25.5767...
\end{align*}
\]

\[\rightarrow \text{26 months}\]
6. Ben now opens up an account with an initial deposit of $2000. The account has an interest rate of 5%/yr compounded daily. He calls this his “soda” account since every day he withdraws $1.50 to buy a soda. If this goes on for two years, how much is in the account at the end of that time?

\[
\begin{align*}
N &= 2 \times 365 \\
I\% &= 5 \\
PV &= -2000 \\
PMT &= 1.50 \\
P/Y &= C/Y = 365 \\
FV &= 1058.79
\end{align*}
\]

7. When Ben graduates from high school, he decides to take $1000 of his money and open a savings account. He has 3 banks which he is looking at. Second National Bank offers an interest rate of 9.43%/yr compounded monthly, First American Bank offers an annual interest rate of 9.52% compounded semi-annually, and Wells Fargo offers an interest rate of 9.38%/yr compounded weekly. Which account should Ben choose?

\[
\begin{align*}
\text{SNB: } &\text{ Eff} \left( 9.43, 12 \right) = 9.8484 \% \Rightarrow \text{ highest} \\
\text{FAB: } &\text{ Eff} \left( 9.52, 2 \right) = 9.7466 \% \\
\text{WF: } &\text{ Eff} \left( 9.38, 52 \right) = 9.8247 \%
\end{align*}
\]

Choose SNB
8. If Ben had invested his $1000 in an account which compounded interest continuously at a rate of 9.88%/yr, how much would be in the account after 4 years?

\[ F = Pe^{rt} = 1000e^{(0.0988 \times 4)} \approx \$1484.68 \]
9. Ben is planning for the near future. In 6 years, he wants to take a vacation to New York City. He knows that he will need $10,000 for this vacation.

(a) How much should he deposit each quarter into a sinking fund he has set up if this account earns interest at a rate of 5.5%/yr compounded quarterly and he opens the account with $1000? How much total interest will be earned?

N = 24
I% = 5.5
PV = -1000
FV = 10000
P/Y = C/Y = 4

What was put in? (305.32)(24) + 1000 = 8327.68
Interest = 10000 - 8327.68 = $1672.32

(b) If Ben doesn’t make regular deposits, how much does he need to have in the account now to accumulate $10,000 in 6 years? How much interest would be earned in this case?

N = 24
I% = 5.5
PV = ?
FV = 10000
P/Y = C/Y = 4

PV = $7205.42
Interest = 10000 - 7205.42 = $2794.58
10. When Ben enters college, he takes out a student loan for $20,000. When he graduates, he will have to make semiannual payments for 15 years with the loan accruing interest at a rate of 2.2%/yr compounded semiannually.

(a) What will Ben’s semiannual payment be?

\[ N = 30 \quad I% = 2.2 \quad PV = 20000 \quad \text{PMT} = ? \]

\[ \text{P/Y} = \text{C/Y} = 2 \quad \text{FV} = 0 \]

\[ \text{Semiannual Payment} = \$786.33 \]

(b) If Ben can pay $850 each semiannual period, in how many years will he pay off the loan?

\[ N = ? \quad PMT = -850 \quad PV = 20000 \quad \text{P/Y} = \text{C/Y} = 2 \quad \text{FV} = 0 \]

\[ N = 27.3783 \ldots \]

\[ \frac{28 \text{ periods}}{2} = 14 \text{ years} \]
11. Once Ben has graduated from college and wisens up, he gets a good job and opens up a retirement account to prepare for the long-term future. At the end of each month he will put $325 of his paycheck into a retirement account that earns interest at a rate of 4%/yr compounded monthly.

(a) How much money will be in the retirement account when Ben retires in 50 years?
\[
N = 50(12) \quad PMT = -325 \quad FV = 620540.83
\]

(b) When Ben retires and no longer makes deposits, he will begin to withdraw monthly payments from this same account for the estimated remaining 15 years of his life. How much will he get each month in retirement, assuming the ending balance in the account is $0?
\[
N = 15(12) \quad PMT = ? \quad PV = -620540.83 \quad FV = 0 \quad P/Y = C/Y = 12
\quad \text{Solution: } 4590.07
\]

(c) If Ben wants to receive $5000 monthly payments instead for his 15 years of retirement, how much needs to be in the account when he retires?
\[
N = 15(12) \quad PMT = 5000 \quad FV = 0 \quad PV = ? \quad P/Y = C/Y = 12
\quad \text{Solution: } 675960.74
12. Suppose Ben wins the lottery (not likely). He has the option of accepting a $400,000 lump sum or accepting yearly payments of $54,000 for 10 years. The lottery commission will make these payments from an account that earns interest at a rate of 5%/yr compounded annually. How much do they need to deposit in this account now so that the lottery payments can be made? Should you accept the lump sum or the payments?

\[ N = 10 \quad \text{PMT} = 54000 \quad \text{PV} = 416,973.69 \]

\[ I% = 5 \quad \text{FV} = 0 \quad \text{P/Y=C/Y} = 1 \]

Take payments here b/c has higher PV.
13. Ben goes out and purchases a car. He provides a down payment of $2000 and takes out a loan on the remaining balance. In order to pay off the car, he will have to pay $250 a month for 5 years at an interest rate of 10%/yr compounded monthly.

(a) What was the cash price of the car?

\[
\begin{align*}
N &= 5 \times 12 \\
PMT &= -250 \\
P &= 10 \\
FV &= 0 \\
PV &= ? \\
P/Y &= C/Y = 12
\end{align*}
\]

\[\text{PV} = \text{value of loan} = $11766.34 \]

\[\text{PD of 2000} \]

\[\text{Cash price} = \$13766.34\]

(b) How much will he end up paying in interest?

\[\text{Pay on loan: } 250(5)(12) = 15000 \]

\[\text{Interest paid on loan: } 15000 - 11766.34 = \$3233.66\]

(c) What is his outstanding principal after 3 years? What is his equity?

\[N = \# \text{ of remaining payments} \rightarrow \text{Solve for PV.}\]

\[
\begin{align*}
N &= 2 \times 12 \\
PMT &= -250 \\
P &= 10 \\
FV &= 0 \\
PV &= ? \\
P/Y &= C/Y = 12
\end{align*}
\]

\[\text{OP} = \$5417.71\]

\[\text{Equity} = \text{Value of Item} - \text{OP} = \$13766.34 - 5417.71 = \$8348.63\]