Math 166 Week in Review 7
Sections F.1-F.4a

1. When Ben is in 2nd grade, his parents open up a bank account for him that earns interest at a simple interest rate of 8.5% per year. Ben puts $20 in the account to begin with. His goal is to have $71 in his account so he can buy a Gameboy. How long will it take to have $71 in the account?

\[ \text{PV} = \text{PMT} = \]  
\[ \text{FV} = \text{I} = \text{r}t \]  
\[ f = P(1 + rt) \]

\[ r = 0.085 \]  
\[ P = 20 \]  
\[ F = 71 \]  
\[ I = 51 \]  
\[ I = Prt \]  
\[ 51 = 20(0.085)t \]  
\[ t = 30 \text{ yrs} \]
2. Ben doesn’t have this kind of time, so he decides to open up a different bank account with a better simple interest rate of 11%. If Ben wants to have $71 in 15 months, how much should he invest in this new account?

\[
\begin{align*}
N &= \text{ } \\
I\% &= \text{ } \\
PV &= \text{ } \\
FV &= \text{ } \\
P/Y = C/Y &= \text{ } \\
\end{align*}
\]

\[
\begin{align*}
F &= 71 \\
r &= 0.11 \\
t &= \frac{15}{12}\text{ yrs} \\
P &= ? \\
F &= P(1 + rt) \\
71 &= P\left(1 + 0.11 \cdot \frac{15}{12}\right) \\
P &= \frac{71}{1 + 0.11 \cdot \frac{15}{12}} \\
P &= \$62.42
\end{align*}
\]
3. Ben doesn’t have this kind of money, so he changes accounts again. This time he starts his account with $30 (his parents helped out) and he plans on just leaving it there for a while. In 50 months, Ben has $39.25 in his account. What was the simple interest rate on this account?

\[
P = 30 \\
t = \frac{50}{12} \text{ yr} \\
F = 39.25
\]

\[
I = 9.25
\]

\[
I = Prt
\]

\[
9.25 = 30 \cdot r \cdot \frac{50}{12}
\]

\[
r = 0.074
\]

\[
7.4\%
\]
4. Ben is now in 7th grade and is becoming smarter. He decides that simple interest rates aren’t good. So he takes his $209 birthday money and puts it in a savings account that earns interest at a rate of 9%/year compounded quarterly.

(a) How much will Ben have at the end of 3 years?

\[ N = 3 \times 4 = 12 \quad \text{PMT} = 0 \]
\[ \text{I}\% = 9 \quad \text{FV} = ? \]
\[ \text{PV} = -209 \quad \text{P/Y=C/Y} = 4 \]

\[ \text{FV} = \$272.96 \]

(b) How much interest is earned in total?

\[ 272.96 - 209 = \$63.96 \]

(c) How much interest is earned in the 3rd quarter of the 3rd year?

\[ N = 10 \quad \text{PMT} = 0 \quad \text{FV after 10 quarters:} \]
\[ \text{I}\% = 9 \quad \text{FV} = ? \]
\[ \text{PV} = -209 \quad \text{P/Y=C/Y} = 4 \]

\[ \text{FV after 11 quarters:} \]
\[ N = 11 \quad \text{PMT} = 0 \]
\[ \text{I}\% = 9 \quad \text{FV} = ? \]
\[ \text{PV} = -209 \quad \text{P/Y=C/Y} = 4 \]

\[ \text{Interest:} \quad 266.96 - 261.08 = \$5.88 \]

(d) What would the interest rate have to be if he wants to have $300 at the end of the 3 years and interest is still compounded quarterly?

\[ N = 3 \times 4 = 12 \quad \text{PMT} = 0 \]
\[ \text{I}\% = ? \quad \text{FV} = 300 \]
\[ \text{PV} = -209 \quad \text{P/Y=C/Y} = 4 \]

\[ \text{(Answer)} \]
5. After 10th grade, Ben gets a part time job. At the end of each month he deposits $125 from his paycheck into a savings account that has an interest rate of 8.65% compounded monthly.

(a) How much is in the account after 2 years?

\[
\begin{align*}
N &= 2 \times 12 \\
PMT &= -125 \\
I% &= 8.65 \\
FV &= ? \quad \text{(3262.35)} \\
PV &= 0 \\
P/Y = C/Y &= 12
\end{align*}
\]

(b) How much total interest was earned?

What was put in? \((125)(2)(12) = 3000\)

\[
\text{Interest: } 3262.35 - 3000 = \boxed{262.35}
\]

(c) How much interest was earned in the 3rd month of the second year?

\[
\begin{align*}
N &= 14 \\
PMT &= -125 \\
I% &= 8.65 \\
FV &= \boxed{1834.41} \\
PV &= 0 \\
P/Y = C/Y &= 12
\end{align*}
\]

\[
\begin{align*}
N &= 15 \\
PMT &= -125 \\
I% &= 8.65 \\
FV &= \boxed{1972.63} \\
PV &= 0 \\
P/Y = C/Y &= 12
\end{align*}
\]

Account increased by: \(1972.63 - 1834.41 = \boxed{138.22}\), but $125 of this was a deposit.

So, interest = \(138.22 - 125 = \boxed{13.22}\)

(d) In how many months will Ben have $3500 in this account?

\[
\begin{align*}
N &= ? \\
PMT &= -125 \\
I% &= 8.65 \\
FV &= 3500 \\
PV &= 0 \\
P/Y = C/Y &= 12
\end{align*}
\]

\(N = 25.596... \rightarrow 26\) months
6. When Ben graduates from high school, he decides to take $1000 of his money and open a savings account. He has 3 banks which he is looking at. Second National Bank offers an interest rate of 9.43%/yr compounded monthly, First American Bank offers an annual interest rate of 9.52% compounded semi-annually, and Wells Fargo offers an interest rate of 9.38%/yr compounded weekly. Which account should Ben choose?

\[
\begin{align*}
\text{SNB: } \text{Eff}(9.43, 12) &= 9.8484\% \\
\text{FAB: } \text{Eff}(9.52, 2) &= 9.7466\% \\
\text{WF: } \text{Eff}(9.38, 52) &= 9.8247\%
\end{align*}
\]

**Highest**

Choose SNB

7. If Ben had invested his $1000 in an account which compounded interest continuously at a rate of 9.88%/yr, how much would be in the account after 4 years?

\[
F = Pe^{rt}
\]

- \( P = 1000 \)
- \( r = 0.0988 \)
- \( t = 4 \) yrs

\[
F = 1000e^{0.0988 \cdot 4} = \text{\$1484.68}
\]
8. Ben is planning for the near future. In 6 years, he wants to take a vacation to New York City. He knows that he will need $10,000 for this vacation.

(a) How much should he deposit each quarter into a sinking fund he has set up if this account earns interest at a rate of 5.5%/yr compounded quarterly and he opens the account with $1000? How much total interest will be earned?

\[ N = 6 \times 4 = 24 \quad \text{PMT} = ? \]
\[ I\% = 5.5 \quad FV = 10000 \]
\[ PV = -1000 \quad P/Y = C/Y = 4 \]

Put in: \((305.32)(24) + 1000 = \$8327.68\)

Interest = 10000 - 8327.68 = \$1672.32

(b) If Ben doesn’t make regular deposits, how much does he need to have in the account now to accumulate $10,000 in 6 years? How much interest would be earned in this case?

\[ N = 24 \quad \text{PMT} = 0 \]
\[ I\% = 5.5 \quad FV = 10000 \]
\[ PV = ? \quad P/Y = C/Y = 4 \]

\[ \text{Interest} = 10000 - 7205.42 = \$2794.58 \]
9. When Ben enters college, he takes out a student loan for $20,000. When he graduates, he will have to make semiannual payments for 15 years with the loan accruing interest at a rate of 2.2%/yr compounded semiannually.

(a) What will Ben’s semiannual payment be?

\[
\begin{align*}
N &= 30 \\
I\% &= 2.2 \\
PV &= 20000 \\
FV &= 0 \\
P/Y = C/Y &= 2 \\
\text{PMT} &= \frac{786.33}{2} \\
\end{align*}
\]

(b) If Ben can pay $850 each semiannual period, in how many years will he pay off the loan?

\[
\begin{align*}
N &= \frac{28}{2} = 14 \text{ yrs} \\
\text{PMT} &= -850 \\
I\% &= 2.2 \\
PV &= 20000 \\
FV &= 0 \\
P/Y = C/Y &= 2 \\
\end{align*}
\]

\(N = 27.378 \ldots \rightarrow 28 \text{ semiannual periods.}\)
10. Once Ben has graduated from college and wisdom up, he gets a good job and opens up a retirement account to prepare for the long-term future. At the end of each month he will put $325 of his paycheck into a retirement account that earns interest at a rate of 4%/yr compounded monthly.

(a) How much money will be in the retirement account when Ben retires in 50 years?

\[ \begin{align*}
N &= 60 \cdot 12 \\
PMT &= -325 \\
I\% &= 4 \\
FV &= ? \\
PV &= 0 \\
P/Y &= C/Y = 12
\end{align*} \]

\[ \$620,540.83 \]

(b) When Ben retires and no longer makes deposits, he will begin to receive monthly payments from this same account for the estimated remaining 15 years of his life. How much will he get each month in retirement?

\[ \begin{align*}
N &= 15 \cdot 12 \\
PMT &= ? \\
I\% &= 4 \\
FV &= 0 \\
PV &= 620,540.83 \\
P/Y &= C/Y = 12
\end{align*} \]

\[ \$4590.07 \]

(c) If Ben wants to receive $5000 monthly payments instead for his 15 years of retirement, how much needs to be in the account when he retires?

\[ \begin{align*}
N &= 15 \cdot 12 \\
PMT &= -5000 \\
I\% &= 4 \\
FV &= 0 \\
PV &= ? \\
P/Y &= C/Y = 12
\end{align*} \]

\[ \$675,960.74 \]
11. Suppose Ben wins the lottery (not likely). He has the option of accepting a $400,000 lump sum or accepting yearly payments of $54,000 for 10 years. The lottery commission will make these payments from an account that earns interest at a rate of 5% yr compounded annually. How much do they need to deposit in this account now so that the lottery payments can be made? Should you accept the lump sum or the payments?

\[
\begin{align*}
N &= 10 \\
I% &= 5 \\
\text{PMT} &= -54000 \\
FV &= 0 \\
\text{PV} &= ? \\
P/Y &= C/Y = 1
\end{align*}
\]

Choose one w/ Higher PV.

Choose payments

\$416,973.69
12. Ben goes out and purchases a car. He provides a down payment of $2000 and takes out a loan on the remaining balance. In order to pay off the car, he will have to pay $250 a month for 3 years at an interest rate of 10%/yr compounded monthly.

(a) What was the cash price of the car?

\[ \text{Value of a loan} = \text{Cash price} - \text{down payment}. \]
\[ 11766.34 = \text{Cash price} - 2000 \]
\[ \text{Cash price} = 11766.34 + 2000 = 13766.34 \]

(b) How much will he end up paying in interest?

\[ \text{pay on loan:} (250)(5)(12) = 15000 \] [out of pocket on loan]

\[ \text{Value of loan:} 11766.34 \]
\[ \text{Interest} = 15000 - 11766.34 = 3233.66 \]

(c) What is his outstanding principal after 3 years? What is his equity?

\[ N = \# \text{ of remaining payments} \]
\[ = 2 \cdot 12 = 24 \]
\[ \text{N= 24} \quad \text{PMT= } -250 \]
\[ \text{I%= 10} \quad \text{FV= 0} \]
\[ \text{PV= ?} \quad \text{P/Y=C/Y= 12} \]

\[ \text{Equity} = \text{Value of item} - \text{Outstanding Principal} \]
\[ = \frac{13766.34}{5417.71} = 8348.63 \]