Math 166 Exam 2 Review
Sections 2.1-2.4 & 3.1-3.4

Note: Although this review highlights all sections covered on the exam, it is slightly more heavily weighted on the new material this week: Sections 3.2-3.4. For additional practice problems on previous material, please take a look at Week in Reviews 4 and 5.

1. Calculate the following probabilities. \( Z \) is the standard normal random variable.

(a) \( P(-0.33 < Z < 0.47) \)

\[
\text{normalcdf}(-0.33, 0.47, 0, 1) = 0.3101
\]

(b) \( P(Z \leq 0.93) \)

\[
\text{normalcdf}(-1.899, 0.93, 0, 1) = 0.8238
\]

2. Find the value of \( a \) given the following probabilities.

(a) \( P(Z < a) = 0.4542 \)

\[a = \text{invNorm}(0.4542, 0, 1) = -0.1151\]

(b) \( P(Z \geq a) = 0.9778 \)

\[a = \text{invNorm}(0.0222, 0, 1) \approx -2.0103\]

(c) \( P(-a \leq Z < a) = 0.5254 \)

\[a = \text{invNorm}(0.7627, 0, 1) \approx 0.7150\]
3. Suppose that the weights of students at a university are normally distributed with a mean of 165 lb and a standard deviation 24 lb. What is the probability that a student selected at random
(a) weighs between 150 and 200 pounds inclusive?

\[ \text{Normalcdf}(150,200,165,24) \approx 0.6616 \]

(b) weighs more than 215 pounds?

\[ \text{Normalcdf}(215,1699,165,24) \approx 0.0186 \]

(c) What weight corresponds to the 70th percentile?

\[ a = \text{invNorm}(0.7, 165, 24) \approx 177.5856 \text{ lb} \]

4. The government of a certain country wants to create a system for tax purposes where families are classified as “elite,” “upper class,” “middle class,” or “lower class” based on the total family income. Suppose family incomes in this country are normally distributed with a mean of $30,000 and a standard deviation of $9,000. If this government knows that they want 15% elite, 25% upper class, 40% middle class, and 20% lower class, what would be the range of income classified as “middle class”?

\[ A = \text{invNorm}(0.2, 30000, 9000) \approx \$22425.41 \]

\[ B = \text{invNorm}(0.6, 30000, 9000) \approx \$32280.12 \]

Between $22425.41 and $32280.12.
5. There are 130 boxes of Cheerios in a grocery store. The number of Cheerios was counted in each box. The data is below.

<table>
<thead>
<tr>
<th>Number of Cheerios</th>
<th>510</th>
<th>480</th>
<th>467</th>
<th>434</th>
<th>521</th>
<th>535</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Boxes</td>
<td>23</td>
<td>9</td>
<td>17</td>
<td>30</td>
<td>40</td>
<td>11</td>
</tr>
</tbody>
</table>

Find the mean, median, mode, standard deviation, and variance for the number of Cheerios in a box.

\[ \bar{X} = 490.2615 \]

\[ \text{Med} = 510 \]

\[ \text{Mode} = 521 \quad \text{(value with highest frequency)} \]

\[ \sigma = 36.5723 \]

\[ \text{Variance} = \sigma^2 = 1337.5316 \]

6. Consider the following probability distribution.

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.2</td>
<td>0.12</td>
<td>0.3</td>
<td>0.22</td>
<td>0.16</td>
</tr>
</tbody>
</table>

(a) Fill in the missing probability and draw a probability histogram for \( X \).

(b) Find \( E(X) \), \( \sigma \), \( \text{Var}(X) \), and the mode.

\[ E(X) = \bar{X} = 3.02 \]

\[ \sigma = 1.3340 \]

\[ \text{Var}(X) = \sigma^2 = 1.7796 \quad \text{Mode} = 3 \]

(c) Find \( P(2 \leq X < 5) \).

\[ X = 2, 3, 4 \quad \rightarrow \quad .12 + .3 + .22 = 0.64 \]
7. A game costs $3 to play. The game involves drawing two cards at random from a deck of cards. If a pair of aces is drawn, you win $38. If any other pair is drawn, then you win $13. If two cards of the same suit are drawn, you win $8. Otherwise, you win nothing. Let \( X \) be the net winnings for a person who plays this game.

(a) Find the probability distribution of \( X \).

\[
\begin{array}{c|c|c|c}
X & \text{2 Aces} & \text{Other pair} & \text{Same suit} & \text{Lose} \\
38 - 3 = & 12 \cdot \frac{C(4, 2)}{C(52, 2)} & \frac{12}{221} & \frac{4 \cdot C(13, 2)}{C(52, 2)} & \frac{12}{17} \\
13 - 3 = & \frac{12}{221} & 4 \cdot \frac{C(13, 2)}{C(52, 2)} & \frac{12}{17} & \frac{1}{221} \\
8 - 3 = & \frac{5}{221} & \frac{4}{17} & \frac{12}{17} & \frac{1}{221} \\
0 - 3 = & \frac{35}{221} & \frac{12}{221} & \frac{4}{17} & \frac{1}{221} \\
\end{array}
\]

\[
1 - \left[ \frac{1}{221} + \frac{12}{221} + \frac{4}{17} \right] = \frac{12}{17}
\]

(b) What are the expected net winnings (rounded to the nearest cent)? Is this game fair?

\[
E(X) = 38 \left( \frac{1}{221} \right) + 10 \left( \frac{12}{221} \right) + 5 \left( \frac{4}{17} \right) - 3 \left( \frac{12}{17} \right) = -$0.24
\]

Not fair since \( E(X) \neq 0 \).

8. A car insurance policy covers damages from a car accident. If you get in a major wreck, the insurance company pays out $3000. If you get in a minor wreck, the insurance company pays out $1000.

(a) Suppose your monthly payment is $110. The probability that you get in a major wreck in a given month is 0.01 and the probability you get in a minor wreck is 0.07. What is the insurance company’s expected gain?

\[
\begin{array}{c|c|c}
X \text{ (Major, Minor, Nothing)} & -2890 & -890 & 110 \\
\text{Prob} \text{ (Major, Minor, Nothing)} & 0.01 & 0.07 & 0.92 \\
\end{array}
\]

\[
E(X) = -2890 \cdot 0.01 - 890 \cdot 0.07 + 110 \cdot 0.92 = $10
\]
(b) Suppose that you provide an extra risk to the insurance company because the probability that you get in a minor wreck jumps to 0.15. (The probability you get in a major wreck stays the same.) What can you expect your minimum monthly payment, $a$, to be?

\[
\begin{array}{c|c|c|c}
X & a-3000 & a-1000 & a \\
\text{Prob} & 0.01 & 0.15 & 0.84 \\
\end{array}
\]

\[
E(X) = 0 - (a-3000)(0.01) + (a-1000)(0.15) + 0.84a = 0
\]

\[
0.01a - 30 + 0.15a - 150 + 0.84a = 0
\]

\[
a = \$180
\]

9. Suppose that in a certain country, the probability a person is over 6-ft tall is 0.58. If a group of 40 people is randomly selected from this country, what is the probability that

(a) Exactly half of them are over 6-ft tall?

\[
P(X = 20) = \text{binompdf}(40, 0.58, 20)
\]

\[
\approx 0.0746
\]

(b) At least 24 of them are over 6-ft tall?

\[
P(X \geq 24) = 1 - P(X \leq 23)
\]

\[
= 1 - \text{binomcdf}(40, 0.58, 23)
\]

\[
\approx 0.4652
\]

(c) More than 15 but fewer than 21 are over 6-ft tall?

\[
P(15 < X < 21)
\]

\[
= \text{binomcdf}(40, 0.58, 20) - \text{binomcdf}(40, 0.58, 15)
\]

\[
\approx 0.1859
\]

(d) How many people in this group can you expect to be over 6-ft tall?

\[
E(X) = np = (40)(0.58) = 23.2 \quad \text{[about 23]}
\]

(e) What is the standard deviation for the number of people in this group over 6-ft tall?

\[
s = \sqrt{npq} = \sqrt{40(0.58)(0.42)} = 3.1215
\]
10. A toy chest contains 9 Micro Machines, 7 Lego blocks, and 3 GI Joes.

(a) Suppose that 6 toys are selected at random from the box.

i. How many samples would contain exactly 3 Lego blocks and exactly 2 GI Joes?

\[
C(7,2) \cdot C(9,1) = 945
\]

ii. What is the probability that exactly 3 Micro Machines or exactly 2 Lego blocks are selected?

\[
\frac{\binom{3}{3} \cdot \binom{3}{6}}{\binom{3}{3} \cdot \binom{3}{6} + \binom{7}{1} \cdot \binom{3}{5} - \binom{3}{3} \cdot \binom{7}{2} \cdot \binom{3}{1}} = \frac{723}{1292}
\]

iii. What is the probability that at least 1 Micro Machine is selected?

\[
P(Z \geq 1 \text{ M}) = 1 - P(\text{no M}) = 1 - \frac{\binom{9}{0} \cdot \binom{6}{6}}{\binom{16}{6}} = \frac{641}{646}
\]

iv. Find the expected number of GI Joes in a sample. Let \( X = \# \text{ of GI Joes} \)

<table>
<thead>
<tr>
<th>( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
</table>

\[
\begin{array}{c|c|c|c|c}
  & (3,6) & (16,6) & (3,1) & (16,5) \\
\hline
0 & \frac{\binom{3}{0} \cdot \binom{16}{6}}{\binom{19}{6}} = 286/969 & \frac{\binom{3}{1} \cdot \binom{16}{5}}{\binom{19}{6}} = 156/323 & 0 & 0 \\
\hline
1 & \frac{\binom{3}{2} \cdot \binom{16}{4}}{\binom{19}{6}} = 65/323 & \frac{\binom{3}{3} \cdot \binom{16}{3}}{\binom{19}{6}} = 20/969 & 0 & 0 \\
\end{array}
\]

\[
E(X) = 0.9474 \quad \text{[can use calc or by hand]}
\]

\( (\text{about 1}) \)
(b) If all 19 toys are to be arranged in a row, in how many ways can this be done? (Assume each toy is distinguishable.)

\[ 19! = 1.2165 \times 10^{17} \]

(c) In how many ways can 5 of the 19 be arranged in a row?

\[ P(19, 5) = 1,395,360 \]

or \[ \frac{19!}{(19-5)!} = 1,395,360 \]

(d) In how many ways can these toys be arranged in a row if each type of toy is grouped together?

\[ \left(9! \cdot 7! \cdot 3!\right) \frac{3!}{3!} = 6,584 \times 10^{10} \]

(e) In how many ways can 9 of these toys be arranged in a row if a Micro Machine must be on each end and the 3 GI Joes must be in the middle?

\[ \frac{9!}{M \cdot 14 \cdot 13 \cdot 3! \cdot 2! \cdot 1! \cdot 12 \cdot 11 \cdot \frac{8!}{7!}} = 1,037,836 \]

(f) If toys of each type are identical, in how many distinguishable ways can all the toys be arranged?

\[ \frac{19!}{9! \cdot 7! \cdot 3!} = 110,853,600 \]

(g) If 25 toys are selected one at a time with replacement from the toy chest, what is the probability that you get a Micro Machine no more than 10 times?

success = Micro Machine

\[ n = 25, \quad p = \frac{9}{19} \]

\[ P(X \leq 10) = \text{binomcdf}(25, \frac{9}{19}, 10) \approx 0.2968 \]
11. A 7-character code consists of 2 letters, followed by 4 digits, followed by another letter.

(a) How many codes are possible?

\[
\frac{26 \times 26 \times 10 \times 10 \times 10 \times 10}{26} = 175,760,000
\]

(b) How many codes are possible if the first letter must be a consonant, the last letter must be a vowel, and no letter or digit can be repeated?

\[
\frac{21 \times 21 \times 10 \times 9 \times 8 \times 7 \times 5}{21 \times 21 \times 10 \times 9 \times 8 \times 7 \times 5} = 12,700,800
\]

(c) How many codes are possible if the code cannot start with M, the first two digits cannot be 0, and no letter or digit can be repeated?

\[
\frac{25 \times 25 \times 9 \times 8 \times 7 \times 24}{25 \times 25 \times 9 \times 8 \times 7 \times 24} = 604,800,000
\]

12. There are 10 freshmen, 12 sophomores, 7 juniors, and 4 seniors in a high school class. The Student Council consists of a President, Vice President, Secretary, a 4-person social subcommittee, and a 3-person prom-planning subcommittee. A student cannot hold more than one position or be on more than one committee. How many Student Councils can be formed if

(a) there are no restrictions.

\[
\frac{33 \times 32 \times 31}{P \times VP \times S} \cdot \frac{C(30,4) \times C(26,3)}{Social \times Prom} = 2.3325 \times 10^{12}
\]

(b) the president and vice president must be seniors, and the prom-planning committee must consist of all juniors.

\[
\frac{4 \times 3}{P \times VP} \cdot \frac{28 \times C(27,4) \times C(7,3)}{S \times Social \times Prom} = 206,388,000
\]
13. Determine the possible values of $X$ for the following random variables and classify the random variable as Finite Discrete, Infinite Discrete, or Continuous.

(a) An experiment consists of rolling a fair 6-sided die 4 times. Let $X$ be the sum of the numbers rolled.

$$X = 4, 5, 6, \ldots, 24$$

Finite Discrete

(b) An experiment consists of drawing cards one at a time without replacement from a standard deck until all four Aces have been drawn. Let $X$ be the number of draws needed.

$$X = 4, 5, \ldots, 52$$

Finite Discrete

(c) A bag of marbles contains 5 reds and 6 blues. An experiment consists of pulling marbles one at a time with replacement until a red marble is pulled. Let $X$ be the number of pulls needed.

$$X = 1, 2, 3, \ldots$$

Infinite Discrete

(d) Let $X$ be the amount of time (in hours) a person watches T.V. on a given Saturday.

$$X = \{x \mid 0 \leq x \leq 24\}$$

Continuous

14. Determine whether the following statements are True or False:

(a) **TRUE** (FALSE) An experiment consists of randomly selecting 5 marbles one at a time without replacement from a bag of 5 red and 6 blue marbles and observing the color of each marble. This is a binomial experiment.

(b) **TRUE** (FALSE) The total area under a probability histogram is equal to 1.

(c) **TRUE** (FALSE) A normal curve with $\mu = 15$ and $\sigma = 5$ has a higher peak than a normal curve with $\mu = 20$ and $\sigma = 10$. smaller $\sigma \rightarrow$ higher peak