1. Suppose in a group of 100 people, it is known that 5 are left-handed. If a group of 15 people is selected at random, what is the probability that exactly 3 of them are left-handed?

\[
\frac{\binom{3}{1} \binom{12}{12}}{\binom{15}{15}} = \frac{3}{\binom{100}{15}} \approx 0.0216
\]

2. In a grocery store, you see 300 boxes of cereal on the shelves. You had a secret tip that 9 of these boxes have a prize in them. However, you only have enough money to buy 20 boxes of cereal. What is the probability that you will get at least 1 prize?

\[
P(\geq 1 \text{ prize}) = 1 - P(\text{no prize}) = 1 - \frac{C(9, 0) C(291, 20)}{C(300, 20)} = 0.4672
\]
3. A 5-card hand is dealt from a standard deck of cards. Find the probability that at least 2 cards are hearts.

\[
P(\geq 2H) = 1 - P(0 \text{ or } 1H) = 1 - \left[ \frac{C(13,0)C(39,5) + C(13,1)C(39,4)}{C(52,5)} \right] = \frac{1223}{3332}
\]

4. A jar has 6 red, 9 blue, and 4 white marbles. If 8 marbles are selected at random, what is the probability that

(a) exactly 5 are the same color?

\[
\frac{C(6,5)C(13,3) + C(9,5)C(10,3)}{C(19,8)} = 0.2228
\]

(b) exactly 3 white or exactly 4 blue are selected?

\[
\frac{C(4,3)C(15,5) + C(9,4)C(10,4) - C(4,3)C(9,4)C(6,1)}{C(19,8)}
\]

\[
n(A \cup B) = n(A) + n(B) - n(A \cap B)
\]
5. Suppose a family of 5 and a family of 4 go to the movies. If these 9 people are randomly assigned to sit in a row of 9 seats, what is the probability that each family is seated together?

Total ways: $9!$

Ways where each family is together: $(5!\cdot 4!)/2!

\[\frac{5!\cdot 4!}{9!} = \frac{1}{63}\]

6. In order to pick a 4-digit pin number you roll a fair 6-sided die four times. What is the probability that the first three numbers are odd?

Total 4-digit codes: $6 \cdot 6 \cdot 6 \cdot 6 = 6^4$

Ways where first 3 are odd: $\frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{6}{6} = 3^3 \cdot 6$

\[\frac{3^3 \cdot 6}{6^4} = \frac{1}{8}\]
7. Determine whether the following experiments are binomial (repeated Bernoulli trials).

(a) Picking 3 cards one at a time with replacement from a deck of cards and observing if the card is a king or not.

Yes

(b) Tossing a coin until a head is tossed.

No, number of trials is not fixed.

(c) Rolling a pair of dice 5 times and recording the sum.

No, not "success" and "failure"

(d) Picking 4 marbles one at a time without replacement from a jar of 3 red and 5 blue marbles and observing the color of the marble.

No, probability of success changes.
Trials not independent.
8. A certain medicine is known to cause nausea in 35% of those who take it. In a group of 15 people who are taking this medicine, what is the probability that:

(a) Exactly 7 of them will have nausea?

\[
P(X = 7) = \binom{15}{7} \cdot 0.35^7 \cdot 0.65^8 = 0.1319
\]

(b) At most 5 of them will have nausea?

\[
P(X \leq 5) = \binom{15}{5} \cdot 0.35^5 \cdot 0.65^{10} = 0.5643
\]

(c) At least 9 will have nausea?

\[
P(X \geq 9) = 1 - P(X \leq 8) = 1 - \binom{15}{8} \cdot 0.35^8 \cdot 0.65^7 = 0.0422
\]

(d) More than 6 but at most 10 will have nausea?

\[
P(6 < X \leq 10) = \binom{15}{6} \cdot 0.35^6 \cdot 0.65^9 + \binom{15}{7} \cdot 0.35^7 \cdot 0.65^8 + \binom{15}{8} \cdot 0.35^8 \cdot 0.65^7 + \binom{15}{9} \cdot 0.35^9 \cdot 0.65^6 + \binom{15}{10} \cdot 0.35^{10} \cdot 0.65^5 = 0.2423
\]
9. Suppose a biased coin is tossed 400 times. The coin is biased such that the probability of tossing a head is 0.70. What is the probability that

(a) Exactly 275 or exactly 300 heads are tossed?

\[
P(X = 275 \text{ or } X = 300) = \text{binompdf}(400, 0.7, 275) + \text{binompdf}(400, 0.7, 300) = 0.0410
\]

(b) More than 300 heads are tossed?

\[
P(X > 300) = 1 - \text{binomcdf}(400, 0.7, 300) = 0.0116
\]

(c) Fewer than 123 tails are tossed?

\[
P(X < 123) = \text{binomcdf}(400, 0.3, 122) = 0.6101
\]

(d) At least 107 but fewer than 205 tails are tossed?

\[
\text{binomcdf}(400, 0.3, 204) - \text{binomcdf}(400, 0.3, 106) = 0.9309
\]
10. For the following random variables, list the values that $X$ can assume and state whether it is finite discrete, infinite discrete, or continuous.

(a) Let $X$ be the number of times it takes for you to hit the bull’s eye while playing darts.

$$X = 1, 2, 3, \ldots$$

Infinite Discrete

(b) Let $X$ be the amount of coffee (in ounces) that a person drinks each week.

$$X = \{x \mid x \geq 0\}$$

Continuous

(c) Cards are drawn one at a time from a standard deck of cards with replacement until a king is drawn. Let $X$ be the number of draws needed.

$$X = 1, 2, 3, \ldots$$

Infinite Discrete

(d) 5 marbles are pulled one at a time with replacement from a jar containing 4 red, 6 blue, and 10 green. Let $X$ be the number of times a red marble is pulled.

$$X = 0, 1, 2, 3, 4, 5$$

Finite Discrete

(e) Marbles are pulled one at a time without replacement from a jar containing 4 red, 6 blue, and 10 green until a blue marble is pulled. Let $X$ be the number of pulls needed.

$$X = 1, 2, 3, \ldots 15$$

Finite Discrete

(f) Let $X$ be the amount of time (in hours) a person sleeps in one week.

$$X = \{x \mid 0 \leq x \leq 168\}$$

Continuous $\frac{24}{168}$

(g) 6 cards are drawn one at a time from a standard deck of cards without replacement.

i. Let $X$ be the number of hearts drawn.

$$X = 0, 1, 2, 3, 4, 5, 6$$

Finite Dis.
11. Two fair 5-sided dice are rolled and the numbers rolled are observed. Let $X$ be the positive difference of the numbers rolled.

(a) Find the sample space for this experiment.

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,3), (4,4), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5)\}$$

(b) Find the probability distribution for $X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob</td>
<td>$\frac{5}{25}$</td>
<td>$\frac{8}{25}$</td>
<td>$\frac{6}{25}$</td>
<td>$\frac{4}{25}$</td>
<td>$\frac{2}{25}$</td>
</tr>
</tbody>
</table>

(c) Draw a histogram of this distribution.

(d) What is $P(1 \leq X < 4)$?

$$X = 1, 2, 3$$

$$= \frac{8}{25} + \frac{6}{25} + \frac{4}{25} = \frac{18}{25}$$
12. In a bag of 20 Starbursts, it is known that 7 are orange. A sample of 3 Starbursts is selected at random from the bag. Let $X$ be the number of oranges selected. Find the probability distribution of $X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prob</strong></td>
<td>$\frac{\binom{7}{0}\binom{13}{3}}{\binom{20}{3}}$</td>
<td>$\frac{\binom{7}{1}\binom{13}{2}}{\binom{20}{3}}$</td>
<td>$\frac{\binom{7}{2}\binom{13}{1}}{\binom{20}{3}}$</td>
<td>$\frac{\binom{7}{3}}{\binom{20}{3}}$</td>
</tr>
</tbody>
</table>

$$= \frac{143}{970}$$  $$= \frac{91}{190}$$  $$= \frac{91}{380}$$  $$= \frac{7}{228}$$
13. The probability that a baseball player gets a hit is 0.22. Suppose this player comes to bat 4 times in a game. Let $X$ be the number of times he gets a hit. Find the probability distribution for $X$.

\[
\begin{array}{c|c|c|c|c|c}
X & 0 & 1 & 2 & 3 & 4 \\
\hline
\text{Prob} & 0.3702 & 0.4176 & 0.1767 & 0.0832 & 0.0023 \\
\end{array}
\]

Success = get a hit

$n = 4$, $p = 0.22$

$P(X = 0) = \text{binompdf}(4, 0.22, 0)$

If $\text{type binompdf}(4, 0.22)$, gives you all of them.