Math 166 Week in Review 5  
Sections 2.3, 2.4, & 3.1

1. Suppose in a group of 100 people, it is known that 5 are left-handed. If a group of 15 people is selected at random, what is the probability that exactly 3 of them are left-handed?

\[
P(\text{exactly 3 left-handed}) = \frac{\binom{3}{5,3} \binom{95}{12}}{\binom{100}{15}}
\]

\[
\approx 0.0216
\]
2. In a grocery store, you see 300 boxes of cereal on the shelves. You had a secret tip that 9 of these boxes have a prize in them. However, you only have enough money to buy 20 boxes of cereal. What is the probability that you will get at least 1 prize?

\[
P(\text{at least 1 P}) = 1 - P(\text{no P}) \\
= 1 - \frac{C(9, 0) \cdot C(291, 20)}{C(300, 20)} \\
\approx 0.4672
\]
3. A 5-card hand is dealt from a standard deck of cards. Find the probability that

(a) Exactly 2 of the cards are 10's and exactly 1 of the cards is a 9.

\[
\frac{\binom{4}{2} \binom{4}{1} \binom{44}{2}}{\binom{52}{5}} \approx 0.0087
\]

(b) Exactly 3 cards are spades or exactly 4 cards are hearts.

\[
\frac{\binom{13}{3} \binom{39}{2} + \binom{13}{4} \binom{39}{1} - \binom{52}{5}}{\binom{52}{5}} = 0.0923
\]

(c) At least 2 cards are aces.

\[
\frac{\binom{4}{2} \binom{48}{3} + \binom{4}{3} \binom{48}{2} + \binom{4}{4} \binom{48}{1} - \binom{52}{5}}{\binom{52}{5}} = 0.0417
\]
4. A jar has 6 red, 9 blue, and 4 white marbles. If 8 marbles are selected at random, what is the probability that

(a) exactly 5 are the same color?

\[
\frac{\binom{6}{5}\binom{13}{3} + \binom{9}{5}\binom{10}{3}}{\binom{19}{8}} = 0.2228
\]

(b) exactly 3 white or exactly 4 blue are selected?

\[
\frac{\binom{4}{3}\binom{15}{5} + \binom{9}{4}\binom{10}{4} - \binom{4}{3}\binom{9}{4}\binom{6}{1}}{\binom{19}{8}} = 0.4690
\]
5. Suppose a family of 5 and a family of 4 go to the movies. If these 9 people randomly sit in a row of 9 seats, what is the probability that each family is seated together?

Total # ways to be seated: $9!$

# ways for each family to be seated together: $rac{5!4!2!}{9!} \cdot \frac{2!}{\text{Arrange within each group}}$

$\frac{5!4!2!}{9!} = \frac{1}{63}$
6. In order to pick a 4-digit pin number you roll a fair 6-sided die four times. What is the probability that the first three numbers are odd?

Total # of ways: \[6 \cdot 6 \cdot 6 \cdot 6 = 6^4\]

Total # of ways for first 3 to be odd:

\[3 \cdot 3 \cdot 3 \cdot 6 = 3^3 \cdot 6\]

\[\frac{3^3 \cdot 6}{6^4} = \frac{1}{8}\]
7. Determine whether the following experiments are binomial (repeated Bernoulli trials).

(a) Picking 3 cards one at a time with replacement from a deck of cards and observing if the card is a king or not.
   
   Yes

(b) Tossing a coin until a head is tossed.
   
   No, # of trials is not fixed.

(c) Rolling a pair of dice 5 times and recording the sum.
   
   No, not 2 outcomes

(d) Picking 4 marbles one at a time without replacement from a jar of 3 red and 5 blue marbles and observing the color of the marble.
   
   No, probability of success changes not independent trials.
8. A certain medicine is known to cause nausea in 35% of those who take it. In a group of 15 people who are taking this medicine, what is the probability that:

(a) Exactly 7 of them will have nausea?

\[ P(X = 7) = \text{binompdf}(15, 0.35, 7) \approx 0.1319 \]

(b) At most 5 of them will have nausea?

\[ P(X \leq 5) = \text{binomcdf}(15, 0.35, 5) \approx 0.5643 \]

(c) At least 9 will have nausea?

\[ P(X \geq 9) = 1 - P(X \leq 8) = 1 - \text{binomcdf}(15, 0.35, 8) = 0.0422 \]

(d) More than 6 but at most 10 will have nausea?

\[ P(6 < X \leq 10) = P(X \leq 10) - P(X \leq 6) = \text{binomcdf}(15, 0.35, 10) - \text{binomcdf}(15, 0.35, 6) = 0.2423 \]
9. Suppose a biased coin is tossed 400 times. The coin is biased such that the probability of tossing a head is 0.70. What is the probability that

(a) Exactly 275 heads are tossed?

\[ P(X = 275) = \text{binompdf}(400, 0.7, 275) = 0.0371 \]

(b) More than 300 heads are tossed?

\[ P(X > 300) = 1 - P(X \leq 300) = 1 - \text{binomcdf}(400, 0.7, 300) \]

(c) Fewer than 123 tails are tossed?

\[ P(X < 123) = P(X \leq 122) = \text{binomcdf}(400, 0.3, 122) \approx 0.6101 \]

(d) At least 107 but fewer than 205 tails are tossed?

\[ P(107 \leq X < 205) = P(X \leq 204) - P(X \leq 106) = \text{binomcdf}(400, 0.3, 204) - \text{binomcdf}(400, 0.3, 106) = 0.9309 \]
10. For the following random variables, list the values that $X$ can assume and state whether it is finite discrete, infinite discrete, or continuous.

(a) Let $X$ be the number of times it takes for you to hit the bull’s eye while playing darts.

$$X = 1, 2, 3, \ldots \quad \text{Infinite Discrete}$$

(b) Let $X$ be the amount of coffee (in ounces) that a person drinks each week.

$$X = \{x \mid x \geq 0\} \quad \text{Continuous}$$

(c) Cards are drawn one at a time from a standard deck of cards with replacement until a king is drawn. Let $X$ be the number of draws needed.

$$X = 1, 2, 3, 4, \ldots \quad \text{Inf. Discrete}$$

(d) 5 marbles are pulled one at a time with replacement from a jar containing 4 red, 6 blue, and 10 green. Let $X$ be the number of times a red marble is pulled.

$$X = 0, 1, 2, 3, 4, 5 \quad \text{Fin. Discrete}$$

(e) Marbles are pulled one at a time without replacement from a jar containing 4 red, 6 blue, and 10 green until a blue marble is pulled. Let $X$ be the number of pulls needed.

$$X = 1, 2, 3, 4, \ldots 14, 15 \quad \text{Finite Discrete}$$

(f) Let $X$ be the amount of time (in hours) a person sleeps in one week.

$$X = \{x \mid 0 \leq x \leq 168\} \quad \text{Continuous}$$

(g) 6 cards are drawn one at a time from a standard deck of cards without replacement.

   i. Let $X$ be the number of hearts drawn.

$$X = 0, 1, 2, 3, 4, 5, 6 \quad \text{Fin Dis.}$$

   ii. Let $X$ be the number of Jacks drawn.

$$X = 0, 1, 2, 3, 4 \quad \text{Fin Dis.}$$
11. Two fair 5-sided dice are rolled and the numbers rolled are observed. Let \( X \) be the positive difference of the numbers rolled.

(a) Find the sample space for this experiment.

\[
\mathcal{S} = \begin{cases} 
(1,1), (1,2), (1,3), (1,4), (1,5), \\
(2,1), (2,2), (2,3), (2,4), (2,5), \\
(3,1), (3,2), (3,3), (3,4), (3,5), \\
(4,1), (4,2), (4,3), (4,4), (4,5), \\
(5,1), (5,2), (5,3), (5,4), (5,5)
\end{cases}
\]

\( X \)'s in blue

(b) Find the probability distribution for \( X \).

\[
\begin{array}{c|c|c|c|c|c}
\text{X} & 0 & 1 & 2 & 3 & 4 \\
\hline
\text{Prob} & \frac{5}{25} & \frac{8}{25} & \frac{6}{25} & \frac{4}{25} & \frac{2}{25}
\end{array}
\]

(c) Draw a histogram of this distribution.

(d) What is \( P(1 \leq X < 4) \)?

\[
X = 1, 2, 3
\]

\[
\frac{8}{25} + \frac{6}{25} + \frac{4}{25} = \frac{18}{25}
\]
12. In a bag of 20 Starbursts, it is known that 7 are orange. A sample of 3 Starbursts is selected at random from the bag. Let \( X \) be the number of oranges selected.

(a) Find the probability distribution of \( X \).

\[
\begin{array}{c|c|c|c|c}
X & 0 & 1 & 2 & 3 \\
\hline
\text{Prob} & \frac{\text{143}}{570} & \frac{91}{190} & \frac{91}{380} & \frac{7}{228} \\
\end{array}
\]

(b) Draw a histogram for \( X \).

(c) What is \( P(X < 2) \)?

\[
P(X = 0) + P(X = 1) = \frac{143}{570} + \frac{91}{190} = \frac{208}{285}
\]
13. A group of people were surveyed asking how many credit cards each had. The data is given below.

<table>
<thead>
<tr>
<th>Number of Credit Cards</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total: 128</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of People</td>
<td>14</td>
<td>35</td>
<td>45</td>
<td>20</td>
<td>11</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Let \( X \) be the number of credit cards a person has. Find the probability distribution for \( X \). What is \( P(X \geq 1) \)?

\[
P(X \geq 1) = 1 - \frac{14}{128} = \frac{114}{128}
\]
14. The probability that a baseball player gets a hit is 0.22. Suppose this player comes to bat 4 times in a game. Let $X$ be the number of times he gets a hit. Find the probability distribution for $X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob</td>
<td>0.3702</td>
<td>0.4176</td>
<td>0.1767</td>
<td>0.0332</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

success = hit; $n = 4$; $p = 0.22$

$P(X = 0) = \text{binompdf}(4, 0.22, 0)$

$P(X = 1) = \text{binompdf}(4, 0.22, 1)$

$\text{binompdf}(4, 0.22)$ will give all 5 cases.