Math 166 Week in Review 4  
Sections 2.1 & 2.2

1. At a sandwich shop there are 4 types of bread, 5 kinds of meat, 3 types of cheese, and 6 types of dressing you can choose from. How many different sandwiches are possible, assuming you choose one of each type of item?

\[
\frac{4 \cdot 5 \cdot 3 \cdot 6}{B \cdot M \cdot C \cdot D} = 360
\]

2. On a 15 question multiple choice test, 5 of the questions have 3 answer choices and 10 questions have 5 answer choices. How many ways are there to complete the test if problems may be left blank?

\[
4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = \binom{4^5}{6^{10}}
\]

5 questions  
10 questions
3. How many 5-digit codes are possible if

(a) there are no restrictions placed on the numbers?

\[ 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100,000 \]

(b) no repetitions are allowed?

\[ \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{\text{not} \ 1^{st}, \text{not} \ 2^{nd}} = 30,240 \]

(c) the last digit must be even?

\[ 10 \cdot 10 \cdot 10 \cdot 10 \cdot \frac{5}{\text{even}} = 50,000 \]

(d) the last digit must be 3 or 7 and no repetitions are allowed?

\[ \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 2}{\text{not last}, \text{not last} \ 1^{st}, \text{last}} = 6048 \]

(e) the last digit must be 2, 4, or 5, the first digit cannot be 0, and no repetitions are allowed?

\[ \frac{8 \cdot 8 \cdot 7 \cdot 6 \cdot 3}{\text{not last}, \text{not last}, \text{not last} \ 1^{st}, \text{last}} = 8064 \]

(f) the first digit must be odd, digits must alternate odd and even, and there can be no repetitions?

\[ \frac{5 \cdot 5 \cdot 4 \cdot 4 \cdot 3}{\text{odd} \ E \ O \ E \ O} = 1200 \]
4. A chef has 20 different recipes that she wants to arrange in her recipe book. She has 9 dessert recipes, 6 entree recipes, and 5 appetizer recipes.

(a) How many ways are there to arrange all 20 recipes in the book?

\[ \frac{20!}{2} = 20! \]

(b) If the book only has room for 8 recipes, in how many ways can they be arranged?

\[ \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}{8!} = P(20, 8) = 5,079,110,400 \]

(c) If all recipes fit in the book and if she wants all the dessert recipes together, all the entree recipes together, and all the appetizer recipes together in the book, in how many ways can this be done?

\[ \frac{5! \cdot 6! \cdot 9! \cdot 3!}{\text{Arrange A group} \cdot \text{Arrange E group} \cdot \text{Arrange D group} \cdot \text{Arrange groups}} = 1,8812 \times 10^{11} \]
5. How many different ways are there to arrange the letters in the phrase ICE CREAM CAKE with no spaces?

\[
\frac{12!}{3! \cdot 3! \cdot 2!} = 6,652,800
\]

6. A grocery store stocker needs to stock a shelf with 7 identical cans of soup, 6 identical boxes of cereal, and 5 identical bottles of ketchup. He is in Math 166, so he starts rearranging all the items in a row in different ways. How many different arrangements are possible?

\[
\frac{18!}{7! \cdot 6! \cdot 5!} = 14,702,688
\]
7. Suppose Cold Stone Ice Cream has 17 different types of toppings you can mix in with your ice cream – 7 are fruit toppings and 10 are candy toppings.

(a) How many ways are there to pick 6 different toppings to put in my ice cream?

\[ C(17, 6) = 12,376 \]

(b) How many ways are there to pick 6 different toppings to put in my ice cream if exactly 2 must be fruit and 4 must be candy?

\[ \frac{C(7, 2) \cdot C(10, 4)}{2 \cdot 4} = 4410 \]

(c) How many ways are there to pick 6 different toppings if at most 1 of them can be candy?

\[ \binom{10}{6} C(7, 6) + \binom{10}{1} C(7, 5) = 217 \]

(d) Jack and Jill go to get ice cream. Jack wants 3 fruit and 2 candy toppings. Jill wants 2 fruit and 5 candy toppings, but wants them all to be different than Jack’s. In how many ways can Jack and Jill select their toppings?

\[ \binom{7}{3} \cdot \binom{10}{2} \cdot \binom{4}{2} \cdot \binom{8}{5} = 529,200 \]
8. A business is hiring 20 new employees from a pool of 40 applicants. They plan to hire 1 executive assistant, 1 manager, 1 human resources representative, 7 assistant managers, and 10 administrative assistants. In how many different ways could these hires be made (assuming everyone is qualified for each position)?

\[
\frac{40 \cdot 39 \cdot 38 \cdot \binom{37}{7} \cdot \binom{30}{10}}{1 \cdot 1 \cdot 1 \cdot \binom{7}{4} \cdot \binom{10}{9}} = 1.8337 \times 10^{19}
\]
9. A bag of marbles consists of 7 red, 9 green, and 10 yellow. Suppose a sample of 6 marbles is selected.

(a) How many samples are possible?
\[ C(26, 6) = 230,230 \]

(b) How many samples would contain exactly 4 reds?
\[ \binom{4}{R} \binom{2}{R} = 5985 \]

(c) How many samples would contain no green marbles?
\[ \binom{10}{G} \binom{6}{G} = 12,376 \]

(d) How many samples would contain exactly 5 green or exactly 4 yellow?
\[ \binom{5}{G} \binom{1}{G} \binom{2}{Y} \quad \text{or} \quad \binom{4}{G} \binom{2}{Y} \]
\[ \binom{9}{G} \binom{7}{G} + \binom{10}{G} \binom{16}{G} = 27,342 \]

(e) How many samples would contain exactly 3 yellow or exactly 2 red?
\[ \binom{3}{Y} \binom{3}{Y} \quad \text{or} \quad \binom{2}{R} \binom{4}{R} \]
\[ \binom{7}{G} \binom{16}{G} + \binom{7}{G} \binom{19}{G} \quad \text{or} \quad \binom{7}{G} \binom{16}{G} \binom{9}{G} \]
\[ = 12,5916 \]

(f) How many samples would contain at least 4 red?
\[ \binom{4}{R} \binom{2}{R} \quad \text{or} \quad \binom{5}{R} \binom{1}{R} \quad \text{or} \quad \binom{6}{R} \binom{0}{R} \]
\[ \binom{7}{G} \binom{19}{G} \binom{9}{G} \quad \text{or} \quad \binom{7}{G} \binom{16}{G} \]
\[ = 6391 \]

(g) How many samples would contain at least 1 green?
\[ \text{Total # samples} - \text{# samples we don't want} \]
\[ \text{Total # samples} - \text{# samples w/0 G} \]
\[ \binom{26}{6} - \binom{9}{0} \binom{17}{6} \]
10. A group of 6 people go to the movies. In how many ways can they sit in a row of 6 seats if:

(a) There are no restrictions. \[ \boxed{6!} = 720 \]

(b) The group consists of 4 males and 2 females and a male must be on each end.
\[ \frac{4}{\text{male}} \times 4 \cdot 3 \cdot \frac{2}{\text{female}} \cdot 1 \cdot \frac{3}{\text{male}} = \boxed{1288} \]

(c) The group consists of 3 couples that each want to sit together.
\[ \frac{(2! \cdot 2! \cdot 2!)}{\text{Arrange each couple}} \times \frac{3!}{\text{Arrange 3 groups}} = 48 \]

(d) The group consists of 3 males and 3 females and these groups want to sit together.
\[ \frac{(3! \cdot 3!)}{\text{Arrange groups}} \cdot 2! = 72 \]

Arranged within each group
11. A zoo has 10 elephants, 11 monkeys, and 9 hippos. Suppose 8 of these animals are to be relocated to another zoo.

(a) If at most 6 elephants must be relocated, in how many ways can this be done?

Total ways = Total ways with 7E or 8E

\[ C(30, 8) - \left[ C(10, 7)C(20, 1) + C(10, 8) \right] = 5,250,480 \]

(b) Suppose that 5 animals, of which exactly 2 must be elephants, are to be moved to the Fort Worth Zoo, and the remaining 3 animals, of which exactly 1 must be an elephant, are to be moved to the Houston Zoo. In how many ways can this be done?

\[ \frac{C(10, 2)}{FW \text{ Zoo}} \cdot \frac{C(8, 1)}{C(17, 2)} \cdot \frac{C(20, 3)}{Hou \text{ Zoo}} = 55,814,400 \]

(c) If 3 of the elephants, 2 of the monkeys, and 3 of the hippos are to be lined up for a parade, how many such arrangements are possible?

\[ \frac{C(10, 3)}{Choose E} \cdot \frac{C(11, 2)}{Choose M} \cdot \frac{C(9, 3)}{Choose H} \cdot 8! = 2.2353 \times 10^{10} \]

(d) If 3 of the elephants, 2 of the monkeys, and 3 of the hippos are to be lined up for a parade, how many such arrangements are possible if animals of each type stay together?

\[ C(10, 3)C(11, 2)C(9, 3) \cdot \frac{3!}{\text{Arrange within each group}} \cdot \frac{2!}{\text{Arrange 3 groups}} \cdot \frac{3!}{\text{Arrange 3 groups}} \]

\[ = 2,395,008,000 \]
12. I have 12 pictures that I want to put in a photo album. If the photo album has slots for 15 pictures, in how many ways can the 12 photos be put into the album?

\[ \binom{15}{12} \cdot 12! = 2.1795 \times 10^{11} \]

Choose 12 spots out of 15 where pictures go

Arrange 12 pictures in 12 spots