Math 166 Exam 1 Review
Sections L.1-L.2, 1.1-1.7
Note: This review is more heavily weighted on the new material this week: Sections 1.5-1.7. For more practice problems on previous material, take a look at Week in Reviews 1 and 2.

1. Consider the following partial probability distribution for an experiment with sample space \{a, b, c, d, e\}.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.1</td>
<td>0.23</td>
<td>0.18</td>
<td>0.34</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Let \(E = \{a, b\}\), \(F = \{b, d, e\}\), and \(G = \{c, d\}\) and suppose \(P(G) = 0.52\).

(a) Fill in the missing probabilities in the table.
\[ P(G) = P(c) + 0.34 = 0.52 \rightarrow P(c) = 0.18 \]

Total probability must be 1: \[ 1 - \left(0.1 + 0.23 + 0.18 + 0.34\right) = P(e) \]
\[ 0.15 = P(e) \]

(b) Calculate \(P(F \cap G)\).
\[ F \cap G = b \]
\[ P(F \cap G) = \frac{2}{5} \]

(c) Calculate \(P(E^c)\).
\[ P(E^c) = 1 - P(E) = 1 - \left[0.1 + 0.23\right] = 0.67 \]

(d) Is this a uniform sample space?

**No, not all outcomes are equally likely.**

2. Suppose for two events \(A\) and \(B\) that \(P(A) = 0.3\), \(P(B^c) = 0.8\). If \(A\) and \(B\) independent events, calculate \(P(A^c \cup B)\).

Since independent,
\[ P(A \cap B) = P(A) \cdot P(B) = (0.3)(0.2) = 0.06 \]

\[ P(A^c \cup B) = b + c + d = 0.76 \]

\[ \alpha + b = 0.3 \]
\[ b + c = 0.2 \]
\[ \alpha + b + c + d = 1 \]
\[ b = 0.06 \]

\[ a = 0.24 \]
\[ b = 0.06 \]
\[ c = 0.06 \]
\[ d = 0.56 \]
3. Suppose \( P(E) = 0.4 \), \( P(F) = 0.5 \), and \( P(E \cup F) = 0.6 \). Calculate the following.

(a) \( P(E \cap F) = \frac{b}{0.3} \)

OR: \( P(E \cup F) = P(E) + P(F) - P(E \cap F) \)
\[
0.6 = 0.4 + 0.5 - P(E \cap F) \\
-0.3 = -P(E \cap F) \\
0.3 = P(E \cap F)
\]

(b) \( P(E^c) = c + d = \frac{0.6}{0.6} \)

OR: \( P(E^c) = 1 - P(E) = 1 - 0.4 = 0.6 \)

(c) \( P(E^c \cup F^c) = P\left[(E \cap F)^c\right] = 1 - P(E \cap F) = \frac{0.7}{0.7} \)

regions: \( a, c, d \)
\[
0.1 + 0.2 + 0.4 = 0.7 
\]

(d) \( P(E \cap F^c) = a = \frac{0.1}{0.1} \)

\( \uparrow \) \( E \) occurs but not \( F \)

(e) Are \( E \) and \( F \) mutually exclusive events?

Since \( P(E \cap F) = 0.3 \), then \( E \) and \( F \) \underline{cannot} occur at some time. Thus, they are not mutually exclusive.
4. A fair 4-sided die and a fair 5-sided die are rolled.

(a) What is the probability that exactly one 4 is rolled?

\[
\begin{align*}
\text{Sum} & \quad 2 & \quad 3 & \quad 4 & \quad 5 & \quad 6 & \quad 7 & \quad 8 & \quad 9 \\
\text{Prob} & \quad \frac{1}{20} & \quad \frac{2}{20} & \quad \frac{3}{20} & \quad \frac{4}{20} & \quad \frac{4}{20} & \quad \frac{3}{20} & \quad \frac{2}{20} & \quad \frac{1}{20}
\end{align*}
\]

(b) Write the probability distribution for the sum of the dice.

(c) What is the probability that exactly one 1 is rolled given that the sum of the two numbers is at most 4?

\[
P(\text{exactly 1} \mid \text{sum } \leq 4) = \frac{4}{6} = \frac{2}{3}
\]

(d) What is the probability that the sum of the two numbers is more than 5 if a 3 is rolled?

\[
P(\text{sum } > 5 \mid 3 \text{ is rolled}) = \frac{4}{8} = \frac{1}{2}
\]

5. The odds of winning a certain lottery are 2 to 90. What is the probability of not winning this lottery?

\[
P(W) = \frac{2}{2 + 90} = \frac{2}{92} \quad P(W^c) = 1 - \frac{2}{92} = \frac{90}{92} = \frac{45}{46}
\]
6. The following table gives data of US employment figures in a certain city in March. All numbers are rounded to the nearest thousand.

<table>
<thead>
<tr>
<th></th>
<th>Employed</th>
<th>Unemployed (Not retired)</th>
<th>Retired</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>High School Diploma or Lower</td>
<td>120</td>
<td>30</td>
<td>10</td>
<td>160</td>
</tr>
<tr>
<td>Some College or Bachelor's Degree</td>
<td>330</td>
<td>20</td>
<td>15</td>
<td>365</td>
</tr>
<tr>
<td>Some Graduate School or Higher</td>
<td>400</td>
<td>10</td>
<td>20</td>
<td>430</td>
</tr>
<tr>
<td>Total</td>
<td>850</td>
<td>60</td>
<td>45</td>
<td>955</td>
</tr>
</tbody>
</table>

(a) Find the probability that a randomly selected person from this city is employed or retired.

\[ P(E \cup R) = \frac{850 + 45}{955} = \frac{895}{955} = \frac{179}{191} \]

(b) Find the probability that a randomly selected person from this city is unemployed or has a high school diploma or lower.

\[ P(U \cup H) = \frac{60 + 160 - 30}{955} = \frac{190}{955} = \frac{38}{191} \]

(c) What are the odds that a person from this city is unemployed?

\[ P(U) = \frac{60}{935} = \frac{12}{191} \text{ odds are 12 to 179} \]

(d) Find the probability that a person was employed, given that they had some college or a bachelor's degree.

\[ P(E | C) = \frac{330}{365} = \frac{66}{73} \]

(e) Find the probability that a person who was unemployed had a high school diploma or lower.

\[ P(H | U) = \frac{30}{60} = \frac{1}{2} \]

(f) Are the events "having some graduate school or higher" and "being unemployed" independent events?

Test: Does \( P(G \cap U) = P(G) \cdot P(U) \)?

\[ \frac{10}{955} \neq \left( \frac{430}{955} \right) \cdot \left( \frac{60}{955} \right) \\
0.0105 \neq 0.0235 \]

\[ \text{NOT independent} \]
7. Use the tree diagram to find the indicated probabilities.

(a) \( P(C|B) = \frac{0.2}{0.6} \times \frac{0.9}{0.1} = 0.54 \)

(b) \( P(A \cap D) = \frac{(0.6)(0.9)}{P(A) \times P(D|A)} = 0.54 \)

(c) \( P(C \cap D) = \frac{0.2}{0.6} \times \frac{0.9}{0.1} = 0.54 \)

(d) \( P(D) = (0.6)(0.9) + (0.4)(0.8) = 0.86 \)

(e) \( P(B \cup D) = (0.4)(0.2) + (0.4)(0.8) + (0.6)(0.9) = 0.94 \)

8. A lie detector correctly indicates that a person is lying 94% of the time. However, 11% of the time, the test incorrectly indicates that a person is lying when they really aren’t. A group of students are asked “Do you like math?” while hooked up to a lie detector. It is known that 45% of these students are actually lying. If the test says a person is not lying, what is the probability that the person really is?

\[
\begin{align*}
\text{P}(L \mid -) &= \frac{\text{P}(L \cap -)}{\text{P}(-)} \\
&= \frac{(0.45)(0.06)}{(0.45)(0.06) + (0.55)(0.89)} \\
&= \frac{54}{1033}
\end{align*}
\]
9. A certain university in Texas admitted 35% of the applicants from Texas, 15% of the applicants from other U.S. states, and 11% of international applicants. There were a total of 10,000 applicants to this university. 7,200 were from Texas, 2,000 were from other U.S. states, and 800 were international applicants.

\[
\begin{align*}
\frac{7200}{10000} &= 0.72 \quad T \\
\frac{2000}{10000} &= 0.2 \quad A \\
\frac{800}{10000} &= 0.08 \quad A^c
\end{align*}
\]

(a) What is the probability that an applicant is not admitted to the university?

\[
P(A^c) = (0.72)(0.65) + (0.2)(0.85) + (0.08)(0.89) = 0.7092
\]

(b) What is the probability that an applicant from a U.S. state other than Texas is not admitted to the university?

\[
P(A^c | O) = \frac{0.85}{0.85} = 1
\]

(c) What is the probability that an applicant who is admitted to the university is an international applicant?

\[
P(I \cap A) = \frac{P(I \cap A)}{P(A)} = \frac{(0.08)(0.11)}{1 - 0.7092} = \frac{22}{727}
\]

10. A small business has 2 delivery trucks. On a given day, the probability that the first breaks down is 0.03 and the probability that the second breaks down is 0.07. Assuming independence, what is the probability that

(a) Both trucks will break down on a given day?

\[
P(T_1 \cap T_2) = (0.03)(0.07) = 0.0021
\]

(b) Exactly 1 truck will break down on a given day?

\[
P(T_1 \cap T_2^c) + P(T_1^c \cap T_2) = 0.0958
\]
11. I have 2 bags of marbles. Bag A has 2 red and 3 blue marbles. Bag B has 1 red and 4 blue marbles. An experiment consists of first selecting a marble from Bag A. If the first marble picked is red, it is transferred to Bag B, and then a second marble is pulled from Bag B. If the first marble is blue, that marble along with all of Bag A is dumped into Bag B and a second marble is picked from Bag B. Draw a tree diagram for this experiment.

![Tree Diagram](image)

12. Let $U$ be the set of all residents of College Station.
   
   $B = \{x \in U | x$ was born in Texas$\}$
   
   $T = \{x \in U | x$ is a student at Texas A&M University$\}$
   
   $A = \{x \in U | x$ is at least 21 years of age$\}$

   (a) Write the set $(B \cap T) \cup (T^c \cap A)$ in words.
   
   The set of all residents of CS who were born in Texas and are students at TAMU or who are not students at TAMU but are at least 21 years old.

   (b) Using set notation, write “the set of all residents of College Station who were born in Texas and are younger than 21 years old, but are not students at Texas A&M University.”

   $B \cap A^c \cap T^c$

13. Shade the set $(A^c \cup B) \cap C^c$ in a Venn diagram.

![Venn Diagram](image)
14. 500 people were surveyed about what Oscar-nominated movies they had seen. 200 of them had seen *The Departed*, 225 of them had seen *Forrest Gump*, and 165 people had seen neither. Fill in the Venn diagram below.

\[
\begin{align*}
\alpha + b &= 200 \rightarrow \alpha = 110 \\
b + c &= 225 \rightarrow b = 90 \\
a + b + c &= 500 \rightarrow c = 135 \\
a + b + c + d &= 500 \rightarrow d = 165
\end{align*}
\]

(a) What is the probability that a person selected at random from this group had not seen *Forrest Gump*? 
\[P(F^c) = \frac{a + d}{500} = \frac{11 + 165}{500} = \frac{11}{20} \]

(b) Are the events “seeing *The Departed*” and “seeing *Forrest Gump*” independent events for this group of people? 
\[P(D \cap F) = P(D) \cdot P(F) \]
\[\frac{90}{500} = \left( \frac{200}{500} \right) \left( \frac{275}{500} \right) \]
\[.18 = .18 \checkmark \]

YES, are independent

15. A certain high school offers 3 AP exams: Calculus, U.S. History, and English. There are 185 students in the senior class. Data is given below of how many seniors took these AP exams.

17 seniors took English and History, but not Calculus
30 seniors took only Calculus
41 seniors took Calculus and History
43 seniors took exactly 2 AP exams
127 seniors took Calculus or English
84 seniors took History
25 seniors took all three AP exams

\[
\begin{align*}
\alpha &= 17 \\
a &= 30 \\
b + e &= 41 \rightarrow b = 16 \\
b + d + f &= 43 \rightarrow d = 10 \\
an + b + d + e + f + q + l + h &= 127 \rightarrow q = 29 \\
b + c + e + f + q + h &= 84 \rightarrow c = 26 \\
e &= 25 \\
a + b + c + d + e + f + q + h &= 185 \rightarrow h = 32
\end{align*}
\]

(a) How many seniors took Calculus but not English?
\[a + b = 146\]

(b) How many seniors took Calculus or did not take History?
\[a + b + d + e + g + h = 142\]

(c) What is the probability that a senior selected at random from this school took Calculus or English, but not History?
\[\frac{a + d + g}{185} = \frac{69}{185}\]
16. The number of Maroon Out shirts sold on a given day is given in the table below by size.

<table>
<thead>
<tr>
<th>Size</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
<th>X-Large</th>
<th>XX-Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>150</td>
<td>225</td>
<td>300</td>
<td>200</td>
<td>75</td>
</tr>
</tbody>
</table>

Total = 950

(a) Write an empirical probability distribution for this data.

<table>
<thead>
<tr>
<th>Size</th>
<th>Small</th>
<th>M</th>
<th>L</th>
<th>XL</th>
<th>XXL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob</td>
<td>(\frac{150}{950})</td>
<td>(\frac{225}{950})</td>
<td>(\frac{300}{950})</td>
<td>(\frac{200}{950})</td>
<td>(\frac{75}{950})</td>
</tr>
</tbody>
</table>

(b) What is the empirical probability that a person who buys a Maroon Out shirt will want a medium or a large?

\[
\frac{225 + 300}{950} = \frac{525}{950} = \frac{21}{38}
\]

17. Let \(U = \{a, b, c, d, e, f, g, h, i, j\}\), \(A = \{a, d, h, j\}\), \(B = \{b, e, f, i\}\), \(C = \{e, h\}\), and \(D = \{a, j\}\).

(a) Determine whether the following statements are True or False.

i. (TRUE) FALSE \(D \subseteq A\)

ii. TRUE (FALSE) \(C \subset B\)

iii. TRUE (FALSE) \(\{h\} \in (A \cup B)\)

iv. TRUE (FALSE) \((A \cap B)^c = U\)

v. TRUE (FALSE) \(\emptyset \in (B \cap D)\)

vi. TRUE (FALSE) \(n(B \cup C) = 6\)

vii. (TRUE) FALSE \(C\) and \(D\) are disjoint sets.

viii. (TRUE) (FALSE) \(B\) has 16 proper subsets. \([H_{95} 157]\)

(b) Calculate the following sets.

i. \((D \cup C)^c \cap A^c\)

\(\left(\{b, c, f, i, j\}\right)^c \cap \{a, d, h, j\} = \{b, c, f, i, j\}\)

ii. \((B \cap C)^c \cup (A \cap D)\)

\(\left\{b, f, i, j\right\} \cup \left\{a, j\right\} = \{a, b, f, i, j\}\)
18. An experiment consists of first randomly selecting a card from a standard deck and observing the suit. If the suit is a heart or club, a coin is tossed observing the side that lands up. If a diamond is drawn, a fair 4-sided die is rolled observing the side that lands up. If a spade is drawn, the experiment ends.

(a) Determine the sample space for this experiment.

\[ S = \{ \text{Ht, Hh, Ct, Ch, D1, D2, D3, D4, S} \} \]

(b) Determine the event \( E \) that a head is tossed.

\[ E = \{ \text{Ht, Ch} \} \]

(c) Determine the event \( F \) that an odd number is rolled.

\[ F = \{ \text{D1, D3} \} \]

(d) Determine the event \( G \) that a black card is drawn.

\[ G = \{ \text{Cl, Ch, S} \} \]

(e) Which pairs of events above are mutually exclusive?

\[ E \cap F = \emptyset \quad F \cap G = \emptyset \]

19. Consider the following propositions (statements):

\( p \): It rained 10 days in August.

\( q \): The average temperature in August is 95\(^\circ\)F.

\( r \): August has the highest monthly average temperature.

\( s \): It rained at least 1 day this month.

(a) Express the following statements in words.

- \( q \land (p \lor \neg r) \)
  The avg temp in August is 95\(^\circ\)F and either it rained 10 days in August or August does not have the highest monthly average temp.

- \( \neg s \)
  It rained no days this month.

- \( s \)
  It did not rain this month.
(b) Express the following compound statements symbolically:

i. August does not have the highest monthly average temperature or an average temperature of 95°, but it did rain 10 days in August.

\[ \sim (r \lor \sim q) \land p \]

ii. It either rained 10 days in August and August does not have the highest monthly average temperature or the average temperature in August is 95°.

\[ (p \land \sim r) \lor \sim q \]

20. Complete truth tables for the following compound statements.

(a) \((\sim p \land q) \lor \sim q\)

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>\sim p</th>
<th>\sim p \land q</th>
<th>\sim q</th>
<th>(\sim p \land q) \lor \sim q</th>
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(b) \((p \lor \sim q) \land \sim (q \lor r)\)

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>\sim q</th>
<th>\sim (q \lor r)</th>
<th>\sim (q \lor r)</th>
<th>(p \lor \sim q) \land \sim (q \lor r)</th>
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