Math 166 Final Exam Review

Note: This review does not cover every concept that could be tested on a final. Please also take a look at previous Week in Reviews for more practice problems. Every instructor makes up their own final, so it would be beneficial for you to also look over your old tests, quizzes, homework, and class notes.

1. Solve the following system of equations.
   \[
   \begin{align*}
   5x + y &= 2x - 13 \\
   3x + 3z &= -18 + 6y \\
   4y - 2x &= 2x + 12
   \end{align*}
   \Rightarrow
   \begin{align*}
   5x + y - 2z &= -13 \\
   3x - 6y + 3z &= -18 \\
   -2x + 4y - 2z &= 12
   \end{align*}
   \]
   \[
   \begin{bmatrix}
   5 & 1 & -2 & -13 \\
   3 & -6 & 3 & -18 \\
   -2 & 4 & -2 & 12
   \end{bmatrix}
   \xrightarrow{\text{Ref}}
   \begin{bmatrix}
   1 & 0 & -3/11 & -32/11 \\
   0 & 1 & -7/11 & 17/11 \\
   0 & 0 & 0 & 0
   \end{bmatrix}
   \]
   \[
   \begin{align*}
   x - \frac{3}{11} z &= -\frac{32}{11} \\
   y - \frac{7}{11} z &= \frac{17}{11}
   \end{align*}
   \Rightarrow
   \begin{align*}
   x &= -\frac{32}{11} + \frac{3}{11} t \\
   y &= \frac{17}{11} + \frac{7}{11} t
   \end{align*}
   \]
   Let \( z = t \).
   \[
   (x, y, z) = \left(-\frac{32}{11} + \frac{3}{11} t, \frac{17}{11} + \frac{7}{11} t, t\right) \text{ where } t \text{ is any real #.}
   \]

2. Solve the following matrix equation for the variables \(a, b, c, \) and \(d\).
   \[
   \begin{bmatrix}
   2 & a \\
   3 & b
   \end{bmatrix}^T + \begin{bmatrix}
   1 & -2 & c \\
   0 & 5 & 8
   \end{bmatrix} = -2 \begin{bmatrix}
   -6 & -5 \\
   -1 & 0
   \end{bmatrix}
   \]
   \[
   \begin{bmatrix}
   2 & 3 \\
   a & b
   \end{bmatrix} + \begin{bmatrix}
   b+8-c & d-2+0 \\
   -20-8 & 0+5+0
   \end{bmatrix} = \begin{bmatrix}
   12 & 10 \\
   -2 & -8
   \end{bmatrix}
   \]
   \[
   \begin{bmatrix}
   2 & 3 \\
   a & b
   \end{bmatrix} + \begin{bmatrix}
   14-c & d-2 \\
   -28 & 5
   \end{bmatrix} = \begin{bmatrix}
   12 & 10 \\
   -2 & -8
   \end{bmatrix}
   \]
   \[
   \begin{bmatrix}
   16-c & d+1 \\
   a-28 & b+5
   \end{bmatrix} = \begin{bmatrix}
   12 & 10 \\
   -2 & -8
   \end{bmatrix}
   \]

3. A house costs $189,000. Ben makes a down payment of $12,000 and secures a loan for the remaining balance. The loan is to be paid off over 25 years at an interest rate of 6% per year compounded monthly.
   (a) What is the required monthly payment?
   \[
   \begin{align*}
   \text{Loan} &= 189,000 - 12,000 = 177,000 \\
   \text{PMT} &= \frac{177,000}{\text{Loan}} \times \frac{1}{25} \times \frac{1}{12} \times \frac{6}{100} \\
   \text{PMT} &= \frac{177,000}{12} \times \frac{1}{12} \times \frac{1}{12} \times \frac{6}{100} \\
   \text{PMT} &= \frac{1140.41}{12} \times \frac{1}{12} \times \frac{1}{12} \times \frac{6}{100}
   \end{align*}
   \]
   (b) How much total interest will be paid on this loan?
   \[
   \text{Total paid on loan} - \text{value of loan} = \frac{1140.41 \times 25 \times 12}{12} - 177,000 = 165,123
   \]
(c) How much of the first payment goes towards the principal of the loan?

\[
\text{To interest: } I = (177000)(\frac{0.06}{12}) = 885 \\
\text{To principal: } 1140.41 - 885 = \boxed{\$255.41}
\]

(d) What is his outstanding principal after 20 years of payments? What is his equity?

\[
\begin{align*}
N &= 60 \\
PMT &= -1140.41 \\
I &= 6 \\
FV &= 0 \\
PV &= ? \\
P/Y &= C/Y = 12 \\
\end{align*}
\]

\[N = \# \text{ of remaining payments} = 5 \times 12\]

\[OP = \boxed{\$58988.35}\]

\[\text{Equity} = \text{Value of Item} - OP = 189000 - 58988.35 = \boxed{\$130,011.65}\]

4. A survey was taken of 300 A&M students asking what video game systems out of Atari (A), Nintendo (N), and Gameboy (G) they had growing up. The following data was found.

45 students only had Nintendo.
156 students had a Gameboy.
73 students had all 3 systems.
11 students had an Atari and a Gameboy but not a Nintendo.
115 students had a Nintendo and a Gameboy.
100 students had exactly one of these systems.
101 students had a Nintendo or an Atari, but not a Gameboy.

\[
\begin{align*}
a + b + c + d + e + f + g + h &= 300 \\
c &= 45 \\
d + e + f + g &= 156 \\
e &= 73 \\
d &= 11 \\
a + f &= 116 \\
a + c + g &= 100 \\
a + b + c &= 101 \\
\end{align*}
\]

(a) How many students did not have an Atari?

\[c + f + g + h = \boxed{160}\]

(b) What is \(n((N \cap A) \cup (G \cap N^c))\) ?

\[n(b_1e_1d_1g_1) = \boxed{145}\]

(c) What is the probability that a student in this group had exactly 2 of these systems?

\[
\frac{b + d + f}{300} = \frac{84}{300} = \frac{7}{25}
\]
5. The odds that an event $E$ occurs are 9 to 11, the odds that an event $F$ occurs are 7 to 5, and the odds that $E$ occurs but not $F$ are 1 to 4. What is the probability that neither $E$ nor $F$ occurs?

\[
P(E) = \frac{9}{9+11} = \frac{9}{20}, \quad P(F) = \frac{7}{7+5} = \frac{7}{12}, \quad P(E \cap F^c) = \frac{1}{1+4} = \frac{1}{5}
\]

\[
a + b = \frac{9}{20} \quad \rightarrow \quad b = \frac{1}{4}
\]

\[
b + c = \frac{7}{12} \quad \rightarrow \quad c = \frac{1}{3}
\]

\[
a = \frac{1}{5}
\]

\[
a + b + c + d = 1 \quad \rightarrow \quad d = \frac{1}{3}
\]

\[
P(E^c \cap F^c) = \frac{13}{60}
\]

6. I have a bag of red, yellow, and green Skittles and M&M’s. The number of each is given in the table below. An experiment consists of reaching into the bag and pulling out a piece of candy.

<table>
<thead>
<tr>
<th></th>
<th>Skittles</th>
<th>M&amp;M’s</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>3</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Yellow</td>
<td>8</td>
<td>14</td>
<td>22</td>
</tr>
<tr>
<td>Green</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>20</td>
<td>35</td>
</tr>
</tbody>
</table>

(a) What is the probability that the chosen piece is yellow or a Skittle? \[
\frac{22+15-8}{35} = \frac{29}{35}
\]

(b) What are the odds that a green piece is not chosen?

\[
P(G^c) = 1 - \frac{6}{35} = \frac{29}{35}
\]

Odds are 29 to 6.

(c) If an M&M is chosen, what is the probability it is red?

\[
P(R | M) = \frac{4}{20} = \frac{1}{5}
\]

(d) What is the probability that a yellow piece of candy is a Skittle?

\[
P(S | Y) = \frac{8}{22} = \frac{4}{11}
\]

Let $E$ be the event that a red candy is drawn. Let $F$ be the event that an M&M is drawn.

(e) Are $E$ and $F$ mutually exclusive? NO, since $E$ and $F$ can occur at the same time. $P(E \cap F) = \frac{4}{35}$

(f) Are $E$ and $F$ independent?

Does $P(E \cap F) = P(E) \cdot P(F)$?

\[
\frac{4}{35} = \left( \frac{1}{35} \right) \left( \frac{20}{35} \right)
\]

Yes, are independent.
7. A bag of Hershey's miniatures contains 10 milk chocolates, 9 Mr. Goodbar's, 7 Krackels, and 8 dark chocolates. A sample of 8 chocolates is taken from the bag.

(a) How many samples contain exactly 3 milk chocolates or exactly 4 dark chocolates?

\[
\binom{10}{3} \binom{5}{1} \text{ or } \binom{4}{4} \binom{10}{4} \binom{1}{1} - \binom{3}{1} \binom{4}{4} \binom{1}{1} = \binom{10}{3} \binom{24}{5} + \binom{8}{4} \binom{36}{4} - \binom{10}{3} \binom{8}{4} \binom{16}{1} 
\]

\[
= 6,012,580
\]

(b) What is the probability that the sample contains at least 2 Krackels?

\[
P(\geq 2 \text{K}) = 1 - P(0 \text{ or } 1 \text{K}) = \\
\frac{0 \text{K}, 8 \text{K}^c + 1 \text{K}, 7 \text{K}^c}{\binom{27}{8} + \binom{27}{7} \binom{1}{1}} \\
\frac{8 \text{K}^c}{\binom{27}{8}} = 0.5353
\]

8. In a certain group of students, it is known that 21% live in the dorms. Further, 67% of those who live in the dorms are freshmen whereas 39% of those who do not live in the dorms are freshmen.

(a) What is the probability that a student in this group who is not a freshman lives in the dorms?

\[
P(D \mid F^c) = \frac{P(D \cap F^c)}{P(F^c)} = \frac{(0.21)(0.33)}{(0.21)(0.33) + (0.79)(0.61)} = \frac{693}{5512}
\]

(b) Are “living in the dorms” and “being a freshman” independent events for this group?

\[
P(D \cap F) \overset{?}{=} P(D) \cdot P(F) \\
(0.21)(0.67) \neq (0.21)[(0.21)(0.67) + (0.79)(0.39)] \\
0.1407 \neq 0.094248
\]

NOT independent
9. A game consists of rolling a pair of fair 6-sided dice. The game costs $2 to play. If a double is rolled, you win $4. If the sum of the dice is 9, you win $7. If exactly one two is rolled, you win $1. Otherwise, you win nothing. Let $X$ be the net winnings of a person who plays this game. Find the expected value of $X$.

\[
\begin{align*}
\text{P(X)} & \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \\
\text{Prob} & \quad \frac{1}{36} \quad \frac{2}{36} \quad \frac{4}{36} \quad \frac{3}{36} \quad \frac{10}{36} \quad \frac{16}{36}
\end{align*}
\]

\[E(X) = 2 \left( \frac{6}{36} \right) + 5 \left( \frac{4}{36} \right) - 1 \left( \frac{10}{36} \right) - 2 \left( \frac{16}{36} \right) = -\$0.28\]

10. The probability that a battery produced at a certain factory lasts for more than 5 hours is 0.31. If a factory produces 400 batteries, what is the probability that

(a) at least 150 of them will last for more than 5 hours?

\[P(X \geq 150) = 1 - P(X \leq 149) = 1 - \text{binomcdf}(400, 0.31, 149) \approx 0.0033\]

(b) more than 100 but fewer than 140 will last for more than 5 hours?

\[P(100 < X < 140) = P(X \leq 139) - P(X \leq 100) = \text{binomcdf}(400, 0.31, 139) - \text{binomcdf}(400, 0.31, 100) = 0.9471\]

11. Consider the following propositions (statements).

$p$: The Aggies are going to the Cotton Bowl.

$q$: The Aggies beat Oklahoma.

$r$: The Aggies beat Kansas.

(a) Express the following in words: $(p \land r) \lor q$

The Aggies are going to the Cotton Bowl and beat Kansas, or they did not beat Oklahoma.

(b) Express the following using logic symbols: “Either the Aggies beat Kansas but are not going to the Cotton Bowl or the Aggies beat Oklahoma and are going to the Cotton Bowl.”

\[(r \land \sim p) \lor (q \land p)\]
12. Ben wants to have $9000 so that he can go on a vacation. He deposits $400 every quarter in a savings account that has an interest rate of 4.9%/yr compounded quarterly.

(a) How long will it take for him to achieve his goal?

\[
\begin{align*}
N &= 20 \text{ quarters or 5 years} \\
PMT &= -400 \\
I\% &= 4.9 \\
FV &= 9000 \\
PV &= 0 \\
P/Y = C/Y &= 4
\end{align*}
\]

(b) How much interest is earned on the account in the 3rd quarter of the 3rd year?

\[
\begin{align*}
FV \text{ after 11 quarters} &= 4679.65 \\
FV \text{ after 10 quarters} &= 4227.86
\end{align*}
\]

Increase of 451.79, but 400 was deposit.

\[
\text{Interest} = 51.79
\]

(c) What would the quarterly deposit need to be to have the $9000 in only 3 years?

\[
\begin{align*}
N &= 3.4 \\
PMT &= ? \\
I\% &= 4.9 \\
FV &= 9000 \\
PV &= 0 \\
P/Y = C/Y &= 4
\end{align*}
\]

13. I have 16 DVDs. 5 are comedies, 6 are dramas, and 5 are action movies.

(a) Suppose my DVD case only holds 9 DVDs. I know that I want 3 comedies, 4 dramas, and 2 action movies in the DVD case. How many ways are there to choose and then arrange 9 DVDs in my DVD case if I want the comedies together, the dramas together, and the action movies together?

\[
\frac{\binom{5}{3} \cdot \binom{6}{4} \cdot \binom{5}{2} \cdot 3! \cdot 4! \cdot 2! \cdot 3!}{\binom{16}{9}} = 2592000
\]

(b) Suppose 2 DVDs are chosen at random from the original 16. Let \( X \) be the number of action movies chosen. Find the probability distribution for \( X \).

\[
\begin{array}{c|ccc}
X & 0 & 1 & 2 \\
\hline
\text{Prob} & \frac{\binom{5}{3} \cdot \binom{11}{2}}{\binom{16}{2}} & \frac{\binom{5}{2} \cdot \binom{11}{1}}{\binom{16}{2}} & \frac{\binom{5}{2}}{\binom{16}{2}} \\
& \frac{11}{24} & \frac{11}{24} & \frac{1}{12}
\end{array}
\]
14. Suppose the weights of elephants are normally distributed with a mean of 14000 pounds and a variance of 9,000,000 pounds.

\[ \sigma = \sqrt{\text{Var}} = 3000 \]

(a) What is the probability that an elephant selected at random weighs more than 16500 pounds?

\[ \text{normalcdf}(16500, 1E99, 14000, 3000) \approx 0.2023 \]

(b) What symmetric interval of weights about the mean make up the middle 60% of elephants?

\[ A = \text{invNorm}(0.2, 14000, 3000) = 11475.1363 \]
\[ B = \text{invNorm}(0.8, 14000, 3000) = 16524.8637 \]

From 11475.1363 lbs to 16524.8637 lbs

15. Let \( U = \{1, 2, 3, 4, 5, 6\} \), \( A = \{1, 3, 5\} \), \( B = \{2, 3, 4\} \), and \( C = \{2, 4, 5\} \). Find the following sets.

(a) \( A^c \cap B \)

\[ A = \{2, 4, 6\} \]
\[ B = \{2, 3, 4\} \]
\[ A^c \cap B = \{3\} \]

(b) \( B \cup (A \cap C)^c \)

\[ A \cap C = \{3, 5\} \]
\[ (A \cap C)^c = \{1, 2, 4, 6\} \]
\[ B \cup (A \cap C)^c = \{1, 2, 3, 4, 6\} \]

(c) TRUE  \( \{3\} \subset (A \cap B) \)

\( A \cap B = \{3\} \)
\( \emptyset \neq \{3\} \subset (A \cap B) \)

(d) FALSE  \( 5 \subset C \)

Notation bad

True: \( 5 \in C \)
\( \emptyset \neq \{5\} \subset C \)
\( \emptyset \neq \{5\} \subset C \)

(e) TRUE  \( 1 \in (B \cup C)^c \)

\( B \cup C = \{2, 3, 4, 5\} \)
\( (B \cup C)^c = \{1, 6\} \)

16. Shade the set \((A^c \cap B) \cup C^c\) in a Venn diagram.
17. A box has 85 bags of Cheetos in it. The number of Cheetos in each bag was counted. The data is given below.

<table>
<thead>
<tr>
<th>Number of Bags</th>
<th>10</th>
<th>13</th>
<th>17</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cheetos</td>
<td>120</td>
<td>115</td>
<td>110</td>
<td>100</td>
<td>95</td>
</tr>
</tbody>
</table>

Find the mean, median, mode, standard deviation, and variance for the number of Cheetos in a bag.

- \( \text{Mean} = 105.1765 \)
- \( \text{Median} = 100 \)
- \( \text{Mode} = 95 \) (X-value with highest frequency)
- \( \sigma = 9.0569 \)
- \( \text{Var} = \sigma^2 = 82.0277 \)

18. An experiment consists of selecting a marble from a jar consisting of 4 red, 7 blue and 3 yellow marbles, observing its color. If the marble is red or blue, a fair 6-sided die is rolled, observing whether the number rolled is even or odd. If the marble is yellow, a fair 6-sided die is rolled, observing whether the number rolled is even or odd.

(a) Draw a tree diagram with probabilities for this experiment.

(b) What is the sample space \( S \) for this experiment?

\[ S = \{ \text{RE, RO, BE, BO, YE, YO} \} \]

(c) Is this a uniform sample space? No, not all outcomes are equally likely.

(d) What is the event \( E \) that a blue marble is drawn or an odd number is rolled?

\[ E = \{ \text{BE, BO, RO, YO} \} \]

(e) What is the probability that a blue marble is not drawn and an even number is rolled?

\[ P(\{ \text{RE, YE} \}) = \left( \frac{4}{14} \right) \left( \frac{2}{3} \right) + \left( \frac{3}{14} \right) \left( \frac{1}{2} \right) = \frac{31}{140} \]
19. Construct a truth table for the statement: \((p \lor \sim q) \land r\)

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(r)</th>
<th>(\sim q)</th>
<th>((p \lor \sim q))</th>
<th>((p \lor \sim q) \land r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
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<td>F</td>
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<td>T</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

20. Identify the possible values for the following random variables and classify each as finite discrete, infinite discrete, or continuous.

(a) An experiment consists of tossing a coin until it lands heads. Let \(X = \) the number of tosses needed.

\[ X = 1, 2, 3, \ldots \]

Infinite Discrete

(b) An experiment consists of tossing a coin 12 times. Let \(X = \) the number of times you toss heads.

\[ X = 0, 1, 2, \ldots, 12 \]

Finite Discrete

(c) An experiment consists of drawing cards without replacement from a standard deck until a heart is drawn. Let \(X = \) the number of draws needed.

\[ X = 1, 2, 3, \ldots, 40 \] [There are 39 non-hearts. You are guaranteed an H by 40th.]

Finite Discrete

21. In how many distinguishable ways can 3 maroon, 5 white, and 7 gray t-shirts be hung up in a closet if t-shirts of the same color are identical?

\[
\frac{15!}{3!5!7!} = \frac{360,360}{360,360}
\]
22. A fish tank has $x$ goldfish, $y$ tetras, and $z$ guppies for a total of 74 fish. There are 12 times as many guppies as goldfish and twice as many tetras as guppies. How many of each type of fish are in the tank?

\[
\begin{align*}
x + y + z &= 74 \\
z &= 12x \\
y = 2z
\end{align*}
\]

\[
\begin{bmatrix}
1 & 1 & 1 \\
-12 & 0 & 1 \\
0 & 1 & -2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
x = 2 \\
y = 48 \\
z = 24
\]

23. Ben invests $500 in an account earning simple interest. After 6 months, he has $523.75 in the account. What is the simple interest rate for this account?

\[
P = 500 \\
t = 6/12 \text{ yrs.} \\
F = 523.75
\]

\[
I = F - P = 23.75 \\
I = Prt \\
23.75 = 500(r)(\frac{6}{12})
\]

\[
r = 0.095 \Rightarrow 9.5\%
\]

24. (a) How much does Ben need to invest in an account now that earns interest at a rate of 5.3%/yr compounded weekly so that he has $7000 at the end of 3 years?

\[
N = 3 \cdot 52 \quad \text{PMT} = 0
\]

\[
I\% = 5.3 \quad FV = 7000 \quad PV = ? \quad P/Y = C/Y = 52
\]

(b) How much of this is interest?

\[
7000 - 5971.46 = 1028.54
\]

(c) What is the effective interest rate on this account?

\[
\text{Eff}(5.3, 52) = 5.4401\%
\]

25. If $A$ is a $2 \times 2$ matrix, $B$ is a $3 \times 2$ matrix, and $C$ is a $2 \times 3$ matrix, determine if the following operations are possible. Give the resulting dimensions if the operation is possible.

(a) $CBA$  
\[
2 \times 3 \times 2 \times 2 = ? \times 2 \times 2
\]

(b) $3B^T A$  
\[
2 \times 3 \times 2 \times 2 = ? \times 2 \times 2 + 2 \times 3
\]

(c) $BA + C$  
\[
2 \times 2 \times 2 \times 2 \times 2 + 2 \times 3
\]
26. A 7-character code consists of 4 letters followed by 3 digits. How many 7-character codes are possible if the 4th character must be a vowel, the first digit cannot be 0, and there can be no repetition of letters or digits.

\[
\begin{array}{c|c|c|c|c}
25 & 24 & 23 & 5 & 9 \\
not 1 & not 1st & not 2nd & not 3rd & not previous \\
\hline
\end{array}
\begin{array}{c|c|c}
9 & 8 & 2 \\
not 0 & not previous & previous \\
\hline
\end{array}
= 44,712,000
\]

27. A system of equations has parametric solution \((x, y, z) = (-8 + 3t, 102 - 4t, t)\). If \(x, y,\) and \(z\) must be non-negative whole numbers, what values of \(t\) will give realistic solutions to the system?

\[
t = 3, 4, 5, \ldots, 25
\]

\(t \leq 3\) makes \(x\) negative
\(t > 25\) makes \(y\) negative

28. Solve the matrix equation below for \(X\). (Assume all matrix sizes are appropriate and any necessary inverses exist.)

\[
XF + XD = X + E
\]

\[
XF + XD - X = E
\]

\[
X(F + D - I) = E
\]

\[
X(F + D - I)(F + D - I)^{-1} = E(F + D - I)^{-1}
\]

\[
X = E(F + D - I)^{-1}
\]

29. If \(P(E) = 0.3, P(F) = 0.6\) and \(P(E \cup F) = 0.75\), find \(P(E \cap F)\).

\[
a + b = 0.3 \rightarrow b = 0.05
\]

\[
b + c = 0.6 \rightarrow c = 0.55
\]

\[
b + c + d = 0.75 \rightarrow d = 0.15
\]

\[
a + b + c + d = 1 \rightarrow a = 0.25
\]

\[
P(E \cap F) = 0.05
\]

30. At a dog show, 4 dogs will get Honorable Mention ribbons, 2 dogs will get Excellent ribbons, 1 dog will get a Superior ribbon, and 1 dog will get the Best in Show ribbon. If there are 15 dogs in this competition, in how many ways can the ribbons be awarded?

\[
\frac{C(15, 4) \cdot C(11, 2) \cdot 9 \cdot 8}{\text{HM} \cdot E \cdot S \cdot B} = 5,405,400
\]
31. Suppose that in a study of Coke, Pepsi, and Dr. Pepper consumers in a certain city, it was found that 65% of those who buy Coke one month will also buy Coke the next month, while 28% will switch to Pepsi, and 7% will switch to Dr. Pepper. Further, 54% of those who buy Pepsi one month will buy it the next month, while 35% will switch to Coke, and 11% will switch to Dr. Pepper. Finally, if a consumer buys Dr. Pepper one month, data shows that 70% will buy it the next month, 20% will switch to Coke, and 10% will switch to Pepsi.

(a) If in a given month, 40% bought Coke, 20% bought Pepsi, and 40% bought Dr. Pepper, what percentage of these consumers will buy each of these brands 4 months later?

\[ X_0 = \begin{bmatrix} .4 \\ .2 \\ .4 \end{bmatrix}, \quad T = \begin{bmatrix} .65 & .35 & .2 \\ .28 & .54 & .1 \\ .07 & .11 & .7 \end{bmatrix} \]

\[ X_4 = T^4 X_0 = \begin{bmatrix} .4407 \\ .3114 \\ .2479 \end{bmatrix} \]

44.07% Coke, 31.14% Pepsi, 24.79% DP

(b) What is the steady-state distribution for this Markov process, if it exists?

\[ T \text{ is regular (some power of } T \text{ has all positive entries), so steady-state exists.} \]

Let \[ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \] be steady-state where \[ x + y + z = 1 \]

\[ TX = X \]

\[ \begin{bmatrix} .65 & .35 & .2 \\ .28 & .54 & .1 \\ .07 & .11 & .7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \]

\[ \begin{align*}
.65x + .35y + .2z &= x \\
.28x + .54y + .1z &= y \\
.07x + .11y + .7z &= z
\end{align*} \]

\[ x + y + z = 1 \]

Solve by ref.

\[ \frac{127}{281} \]

\[ \Rightarrow x = \frac{127}{281}, \quad y = \frac{91}{281}, \quad z = \frac{63}{281} \]

In the long run, \( \frac{127}{281} \) will buy Coke, \( \frac{91}{281} \) will buy Pepsi, and \( \frac{63}{281} \) will buy DP.
32. Determine whether each of the following stochastic matrices is a transition matrix for a Markov process that is regular, absorbing, or neither.

(a) \[
\begin{bmatrix}
0.8 & 1 & 0.4 \\
0.1 & 0 & 0.3 \\
0.1 & 0 & 0.3 \\
\end{bmatrix}
\]
   \(T^+\) has all positive entries \(\Rightarrow\) **Regular**

(b) \[
\begin{bmatrix}
0.5 & 0.5 & 0 \\
0 & 0.3 & 0.6 \\
0 & 0.2 & 0.4 \\
\end{bmatrix}
\]
   \(A\) is an absorbing state. \(B \rightarrow A \checkmark\)
   \(C \rightarrow B \rightarrow A \checkmark\)
   \(\boxed{\text{Absorbing}}\)

(c) \[
\begin{bmatrix}
0.7 & 0.2 & 0 \\
0.3 & 0.8 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]
   \(C\) is absorbing, but can't get there from \(A\) or \(B\).
   \(\text{No power of } T^+ \text{ has all pos. entries.}\)
   \(\boxed{\text{Neither}}\)

33. The following matrix is a transition matrix for an absorbing Markov process with four states \(A, B, C,\) and \(D\). Find the limiting matrix and interpret.

\[
T = \begin{bmatrix}
A & B & C & D \\
1 & 0 & 0.3 & 0.2 \\
0 & 1 & 0.4 & 0 \\
0 & 0 & 0.3 & 0.5 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
A & B & C & D \\
1 & 0 & 3/7 & 29/49 \\
0 & 1 & 4/7 & 20/49 \\
a^{\text{large power}} & 0 & 0 & 0 \\
\end{bmatrix}
\]

If you start in \(C\), the long-term probabilities of being absorbed by \(A\) and \(B\) are \(3/7\) and \(4/7\) respectively.

If you start in \(D\), the probs are \(29/49\) and \(20/49\) of being absorbed by \(A\) and \(B\) respectively.