3.4 The Normal Distribution

All of the probability distributions we have found so far have been for finite random variables. (We could use rectangles in a histogram.)

A probability distribution for a continuous random variable cannot be written in a table. It takes the form of a function called a **probability density function**.

A probability density function \( f \) has the following properties:

1. \( f(x) \geq 0 \) for all values of \( x \).
2. The total area of the graph between \( f \) and the \( x \)-axis is equal to 1.

Finding probabilities for a continuous random variable amounts to finding the area under the graph of the p.d.f. between the given boundaries.

A very important continuous probability distribution is the **normal distribution**.

The graph of a normal distribution is a bell-shaped curve, known as the **normal curve**.

![Graph of Normal Distribution](image)

**Properties of the Normal Curve:**

1. The curve has a peak at \( x = \mu \).
2. The curve is symmetric about the vertical line \( x = \mu \).
3. The area under the curve is 1. (which is true for any probability distribution.)
4. The curve always lies above the \( x \)-axis. It never crosses or touches it.
5. The standard deviation, \( \sigma \), determines the flatness of the curve. (The larger \( \sigma \) is, the flatter the curve is.)

The normal curve that has \( \mu = 0 \) and \( \sigma = 1 \) is called the **standard normal curve**. The random variable corresponding to the standard normal curve is called the **standard normal variable** and is denoted by \( Z \).
Since the normal distribution is so common, the calculator has a built-in feature to calculate probabilities. This feature is \textit{normalcdf} and is found by pressing 2nd \textit{VARS} and then choosing option 2:\textit{normalcdf}.

The values that this command needs are

\textit{normalcdf}(left bound, right bound, \mu, \sigma)

If there is no left bound, we traditionally use $-1\ E\ 99$. (The E is typed on your calculator as 2nd, “comma”.)

If there is no right bound, we use $1\ E\ 99$.

Examples:

1. $P(-0.89 < Z < 1.65)$

2. $P(Z < 0.68)$

3. $P(Z > 1.11)$

4. $P(Z \geq 1.11)$
If, however, you are given the probability or area and are asked to find the boundary value of \( Z \) that gives you this area. For these types of problems, we use the \( \text{invNorm} \) function on the calculator. This function is option 3 after pressing 2nd \( \text{VARS} \).

In order to use the \( \text{invNorm} \) command, you must find the total area to the LEFT of the boundary you are solving for.

In order to find a boundary value \( a \), the values that this command needs are:

\[
a = \text{invNorm}(\text{total area to the left of } a, \mu, \sigma)
\]

Examples: Find the value of \( a \) that satisfies the following probabilities.

1. \( P(Z < a) = 0.9523 \)

2. \( P(Z > a) = 0.8289 \)

3. \( P(-a < Z < a) = 0.7345 \)
We can also use the above commands to calculate probabilities and boundary values for normal variables that are not standard normal.

Applications:

1. According to data for a certain city, the weekly earnings of workers are normally distributed with a mean of $700 and a standard deviation of $60.
   
   (a) What is the probability that a worker selected at random from the city makes more than $805?
   
   (b) What minimum weekly earnings would put you in the top 15% of wage earners?
   
   (c) What symmetric interval of wages about the mean comprises 64% of wage earners?

2. A computer manufacturer has data that the amount of time a computer lasts is normally distributed with a mean of 25 months and a standard deviation of 6 months.
   
   (a) Find the probability that a computer made by this manufacturer lasts between 1 and 2 years.
(b) What computer life-length corresponds to the 90th percentile? 
   (The $n$th percentile means that $n\%$ of all data is below this value.)

3. The scores on a math test are normally distributed with a mean of 70 and a variance of 100. If the instructor wants to assign A’s to 15%, B’s to 25%, C’s to 40%, D’s to 15%, and F’s to 5% of the class, find
   (a) the cutoff grade for an A.

   (b) the cutoff grade for a B.

   (c) the cutoff grade to pass (getting a D or higher).