Math 150 Exam 1 Review Problem Set

Note: This exam review does not cover every topic that could be covered on your exam.
Please take a look at previous Week in Reviews for more practice problems.

(This WIR is more heavily weighted on Sections 2.4-2.8 since there has not been a review on these sections yet.)

1. Graph the following functions by using transformations.

(a) \( f(x) = -\frac{1}{2}|x - 2| \)

(b) \( f(x) = 3\sqrt{x + 3} + 2 \)

2. A function \( f(x) \) is horizontally stretched by a factor of 5, reflected across the \( y \)-axis, vertically stretched by a factor of 5, and then shifted down 4. Write a function \( g(x) \) in terms of \( f(x) \) that represents the resulting graph.

3. Determine whether the following functions are even, odd, or neither.

(a) \( f(x) = x^2 - \sqrt{x} \)

(b) \( f(x) = |x| - 5x^{-4} \)
4. For the quadratic function below, write in standard form, find the vertex of the parabola, and find the maximum or minimum value.
\[ f(x) = -3x^2 - 18x - 31 \]

5. For the quadratic function below, find the maximum or minimum value and state the range.
\[ f(x) = 5x^2 + 6x + 4 \]

6. Suppose \( f(x) = \frac{1}{\sqrt{x^2 + x - 2}} \) and \( g(x) = \frac{\sqrt{x + 3}}{x^2 - 9x + 20} \).

   (a) Find the domain of \( f \).

   (b) Find the domain of \( g \).
(c) Find the domain of $f + g$, $f - g$, and $fg$.

(d) Calculate $(f + g)(2)$ and $(fg)(6)$.

(e) Find $\frac{f}{g}$ and its domain.

7. Let $f(x) = \frac{x}{x + 6}$ and $g(x) = x - 1$.

(a) Find $f \circ g$ and its domain.

(b) Find $f \circ f$ and its domain.
8. Let \( f(x) = 2x^2 - 3x \) and \( g(x) = 2x^3 + x \). Calculate the following. Expand fully to polynomial form.

(a) \( f \circ g \)

(b) \( g \circ f \)

9. Determine whether the following define \( y \) as a function of \( x \). If \( y \) IS a function of \( x \), state whether it is a one-to-one function.

(a) \( y - 5 = 3|x - 2| \)  
(b) \( xy^2 + 3y^2 = x \)  
(c) \( y^3 = -8x \)

10. Find inverse functions for the following.

(a) \( f(x) = \sqrt[3]{x^5 + 9} \)  
(b) \( f(x) = \frac{x}{x + 4} \)
11. (Taken from *Precalculus: Functions and Graphs* by Swokowski/Cole)

A man is in a rowboat that is 2 miles from the nearest point $A$ on a straight shoreline. He wishes to reach his house, which is located at a point $B$ that is 6 miles farther down the shoreline from $A$. He plans to row to a point $P$ that is between $A$ and $B$ and then walk the remainder of the distance. Suppose he can row at a rate of 3 mi/hr and can walk at a rate of 5 mi/hr.

(a) If $T$ is the total time required to reach the house, express $T$ as a function of $x$, where $x$ is the distance from $P$ to $B$.

(b) What is the shortest possible travel time? What distance $x$ will result in the shortest travel time?

12. A very large bottle contains 2000 mL of 10% acid solution. An 80% acid solution is being poured into the bottle at a rate of 10 mL/sec.

(a) Express the concentration $C$ of the bottle as a function of time $t$.

(b) When will the concentration be 60%?
13. Find the average rate of change of the function \( f(x) = \frac{x^2}{x + 1} \) from \( x = 5 \) to \( x = 5 + h \).

14. Solve the following equations.

(a) \( 3x^{1/3} + 2x^{-2/3} - 2x^{-5/3} = 0 \)

(b) \( \frac{1}{x-2} + \frac{11}{3x+1} = \frac{28}{(x^2-4)(3x+1)} \)
(c) $9x^3 - 18x^2 - 4x + 8 = 0$

(d) $\sqrt{4x - 19} + 4 = x$

(e) $16x - 24\sqrt{x} + 9 = 0$
15. Solve the following inequalities.

(a) \( \frac{4}{2x + 3} \geq 1 \)

(b) \(-|6x - 11| + 5 \leq 3\)

16. Simplify the following expression and write without negative exponents:
\[
\left( \frac{25x^4 y^{-2}}{z^6} \right)^{3/2} \left( \frac{y^{-3z}}{x^5} \right)^{-3}
\]
17. Simplify the following expression: \( \sqrt[6]{3^{15}x^{22}y^{14}} \)

18. Find the center and radius of the circle \( x^2 + y^2 + 8x - 10y + 37 = 0 \).

19. Consider the points \((4, 6)\) and \((-6, 2)\).
   
   (a) Find the distance between these points.

   (b) Find the equation of the line that is parallel to the line \( 5x + 8y = 12 \) and passes through the midpoint of the line segment between these points.
20. Consider the function \( f(x) = \sqrt{|x - 2|} + x^4 - 5x^2 + 2x + 3. \)

(a) Find the \( x \)-intercepts of \( f \).

(b) Where is \( f \) decreasing?

(c) What is the range of \( f \)?

(d) Solve the equation \( f(x) = x^{1/3} - 2 \).

21. True or False

(a) TRUE FALSE To rationalize the numerator of \( \frac{\sqrt{x} + 10}{x^2} \), multiply numerator and denominator by \( \sqrt{x} + 10 \).

(b) TRUE FALSE If \( L_1 \) has slope \(-4\) and \( L_2 \) is perpendicular to \( L_1 \), then \( L_2 \) has slope \( 4 \).

(c) TRUE FALSE The equation \( y = -\frac{1}{\sqrt{5}} x^2 + x - 5 \) has exactly one real solution.

(d) TRUE FALSE A graph that is symmetric about the \( x \)-axis and the \( y \)-axis must also be symmetric about the origin.