1. Graph the following functions by using transformations.

\[ f(x) = -\frac{1}{3}|x - 2| \]

Start with \( g(x) = |x| \)

- \( g(x-2) = |x-2| \) \( \rightarrow \) Shift right 2
- \( \frac{1}{2}g(x-2) = \frac{1}{2}|x-2| \) \( \rightarrow \) Vertically shrink graph by \( \frac{1}{2} \).
- \( -\frac{1}{2}g(x-2) = -\frac{1}{2}|x-2| \) \( \rightarrow \) Reflect across \( x \)-axis
(b) \( f(x) = 3 \sqrt{x + 3} + 2 \)

\[ g(x) = \sqrt{x} \]
\[ g(x+3) = \sqrt{x+3} \rightarrow \text{Shift left 3} \]
\[ 3g(x+3) = 3\sqrt{x+3} \rightarrow \text{Vertically stretch graph by 3.} \]
\[ 3g(x+3) + 2 = 3\sqrt{x+3} + 2 \rightarrow \text{Shift up 2} \]
2. A function $f(x)$ is horizontally stretched by a factor of 5, reflected across the $y$-axis, vertically stretched by a factor of 5, and then shifted down 4. Write a function $g(x)$ in terms of $f(x)$ that represents the resulting graph.

$$g(x) = 5f\left(-\frac{1}{5}x\right) - 4$$
3. Determine whether the following functions are even, odd, or neither.

(a) \( f(x) = x^2 - \sqrt[3]{x} \)
\[
f(-x) = (-x)^2 - \sqrt[3]{-x} = x^2 + \sqrt[3]{x}
\]
Neither

(b) \( f(x) = |x| - 5x^{-4} \)
\[
f(-x) = |-x| - 5(-x)^{-4} = |x| - 5x^{-4} = f(x)
\]
EVEN

Even: \( f(-x) = f(x) \rightarrow \) symmetric about y-axis
Odd: \( f(-x) = -f(x) \rightarrow \) symmetric about origin.
4. For the quadratic function below, write in standard form, find the vertex of the parabola, and find the maximum or minimum value.

\[ f(x) = -3x^2 - 18x - 31 \]

\[ f(x) = a(x-h)^2 + k \]

Vertex: \((h, k)\)

\[ f(x) = (-3x^2 - 18x + 27) - 31 \]

\[ = -3(x^2 + 6x + 9) - 31 + 27 \]

\[ = -3(x+3)^2 - 4 \]

Vertex: \((-3, -4)\)

\[ a = -3 < 0 \Rightarrow \text{parabola opens downward} \Rightarrow \text{vertex is a maximum.} \]

Maximum value is \(-4\).
5. For the quadratic function below, find the maximum or minimum value and state the range.

\[ f(x) = 5x^2 + 6x + 4 \]

\[ a = 5 > 0 \Rightarrow \text{parabola opens upward} \Rightarrow \text{vertex is a minimum} \]

Minimum value occurs when \[ x = \frac{-b}{2a} = \frac{-6}{2(5)} = \frac{-6}{10} = \frac{-3}{5} \]

Minimum value is \[ f\left(\frac{-3}{5}\right) = 5\left(\frac{-3}{5}\right)^2 + 6\left(\frac{-3}{5}\right) + 4 \]

\[ = 5\left(\frac{9}{25}\right) - \frac{18}{5} + 4 \]

\[ = \frac{9}{5} - \frac{18}{5} + 4 \]

\[ = 9 - \frac{18}{5} + \frac{20}{5} = \frac{11}{5} \]

Minimum value is \( \frac{11}{5} \)

Range: \( \left[ \frac{11}{5}, \infty \right) \)
6. Suppose \( f(x) = \frac{1}{\sqrt{x^2 + x - 2}} \) and \( g(x) = \frac{\sqrt{x + 3}}{x^2 - 9x + 20} \).

(a) Find the domain of \( f \).

\[ \frac{x^2 + x - 2}{(x+2)(x-1)} > 0 \]

\[ (-\infty, -2) \cup (1, \infty) \]

(b) Find the domain of \( g \).

\[ \frac{x^2 - 9x + 20}{(x-4)(x-5)} \neq 0 \]

\[ x \neq 4, 5 \]

\[ x + 3 \geq 0 \]

\[ x \geq -3 \]

\[ [-3, 4) \cup (4, 5) \cup (5, \infty) \]
(c) Find the domain of \( f + g, f - g, \) and \( fg. \)

\[
[-3, -2) \cup (1, 4) \cup (4, 5) \cup (5, \infty)
\]

(d) Calculate \((f + g)(2)\) and \((fg)(6).\)

\[
(f + g)(2) = f(2) + g(2) = \frac{1}{\sqrt{2^2 + 2 - 2}} + \frac{\sqrt{2 + 3}}{2^2 - 9 \cdot 2 + 20} = \frac{1}{2} + \frac{\sqrt{5}}{6} = \frac{3 + \sqrt{5}}{6}
\]

\[
(fg)(6) = f(6) \cdot g(6) = \left( \frac{1}{\sqrt{6^2 + 6 - 2}} \right) \left( \frac{\sqrt{6 + 3}}{6^2 - 9 \cdot 6 + 20} \right) = \frac{1}{(\sqrt{40})(\sqrt{40})} = \frac{3}{2 \sqrt{40}} = \frac{3}{2 \cdot 4 \sqrt{10}} = \frac{3}{4 \sqrt{10}} = \frac{3 \cdot 10}{40}
\]

(e) Find \( \frac{f}{g} \) and its domain.

\[
\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x^2 + x - 2}}{\sqrt{x+3}} \cdot \frac{\sqrt{x^2 - 9x + 20}}{\sqrt{x^2 + x - 2} \cdot \sqrt{x+3}}
\]

\[
\left( \frac{f}{g} \right)(x) = \frac{x^2 - 9x + 20}{\sqrt{x^2 + x - 2} \cdot \sqrt{x+3}} \quad x \neq -3
\]

Domain: \((-3, -2) \cup (1, 4) \cup (4, 5) \cup (5, \infty)\)
7. Let \( f(x) = \frac{x}{x+6} \) and \( g(x) = x - 1 \).

(a) Find \( f \circ g \) and its domain.

\[
(f \circ g)(x) = f(g(x)) = f(x-1) = \frac{(x-1)}{(x-1) + 6}
\]

\[
= \frac{x-1}{x+5}
\]

Domain of \( g \): \( \mathbb{R} \)

\( f(g(x)) \) is defined when \( x \neq 5 \)

Domain of \( f \circ g \): \( (-\infty, 5) \cup (5, \infty) \)

(b) Find \( f \circ f \) and its domain.

\[
(f \circ f)(x) = f(f(x)) = f\left(\frac{x}{x+6}\right) = \frac{x}{(\frac{x}{x+6}) + 6}
\]

\[
= \frac{x}{(\frac{x}{x+6} + 6)} \cdot \frac{x+6}{x+6} = \frac{x}{x+6(6+6)}
\]

\[
= \frac{x}{7x+36}
\]

Domain of \( f \): \( \{x | x \neq -6\} \)

\( f(f(x)) \) is defined for: \( \{x | x \neq -\frac{36}{7}\} \)

Domain of \( f \circ f \): \( \{x | x \neq -6, -\frac{36}{7}\} \)

\((-\infty, -6) \cup (-6, -\frac{36}{7}) \cup (-\frac{36}{7}, \infty)\)
8. Let \( f(x) = 2x^2 - 3x \) and \( g(x) = 2x^3 + x \). Calculate the following. Expand fully to polynomial form.

(a) \( f \circ g \)

\[
(f \circ g)(x) = f(g(x)) = f(2x^3 + x)
\]

\[
= 2(2x^3 + x)^2 - 3(2x^3 + x)
\]

\[
= 2[4x^6 + 4x^4 + x^2] - 6x^3 - 3x
\]

\[
= 8x^6 + 8x^4 + 2x^2 - 6x^3 - 3x
\]

\[
= \frac{8x^6 + 8x^4 - 6x^3 + 2x^2 - 3x}{8}
\]

(b) \( g \circ f \)

\[
(g \circ f)(x) = g(f(x)) = g(2x^2 - 3x)
\]

\[
= 2(2x^2 - 3x)^3 + (2x^2 - 3x)
\]

\[
= 2[8x^6 - 3(2x^2)^2(3x) + 3(2x^2)(3x)^2 - (3x)^3] + 2x^2 - 3x
\]

\[
= 2[8x^6 - 36x^5 + 54x^4 - 27x^3] + 2x^2 - 3x
\]

\[
= 16x^6 - 72x^5 + 108x^4 - 54x^3 + 2x^2 - 3x
\]

\[
(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3
\]
9. Determine whether the following define $y$ as a function of $x$. If $y$ is a function of $x$, state whether it is a one-to-one function.

(a) $y - 5 = 3|x - 2|$
\[ y = 5 + 3|x - 2| \]
IS a function
NOT one-to-one

(b) $xy^2 + 3y^2 = x$
\[ y^2(x + 3) = x \]
\[ y^2 = \frac{x}{x + 3} \]
\[ y = \pm \sqrt{\frac{x}{x + 3}} \]
NOT a function

(c) $y^3 = -8x$
\[ y = 3\sqrt[3]{-8x} \]
IS a function
IS one-to-one
10. Find inverse functions for the following.

(a) \( f(x) = \sqrt[3]{x^5 + 9} \)
\[
\begin{align*}
y &= \sqrt[3]{x^5 + 9} \\
x &= \sqrt[3]{y^5 + 9} \\
x^3 &= y^5 + 9 \\
x^3 - 9 &= y^5 \\
\sqrt[5]{x^3 - 9} &= y \\
f^{-1}(x) &= \sqrt[5]{x^3 - 9}
\end{align*}
\]

(b) \( f(x) = \frac{x}{x + 4} \)
\[
\begin{align*}
y &= \frac{x}{x + 4} \\
x &= \frac{y}{y + 4} \\
x(y + 4) &= y \\
x y + 4x &= y \\
4x &= y - xy \\
4x &= y(1 - x) \\
\frac{4x}{1 - x} &= y \\
f^{-1}(x) &= \frac{4x}{1 - x}
\end{align*}
\]
A man is in a rowboat that is 2 miles from the nearest point A on a straight shoreline. He wishes to reach his house, which is located at a point B that is 6 miles farther down the shoreline from A. He plans to row to a point P that is between A and B and then walk the remainder of the distance. Suppose he can row at a rate of 3 mi/hr and can walk at a rate of 5 mi/hr.

(a) If $T$ is the total time required to reach the house, express $T$ as a function of $x$, where $x$ is the distance from P to B.

$$d = rt = t = \frac{d}{r}$$

$$T = \frac{d}{3} + \frac{x}{5}$$

$$2^2 + (6-x)^2 = d^2$$

$$4 + 36 - 12x + x^2 = d^2$$

$$x^2 - 12x + 40 = d^2$$

$$\sqrt{x^2 - 12x + 40} = d$$

$$T(x) = \frac{\sqrt{x^2 - 12x + 40}}{3} + \frac{x}{5}$$

(b) What is the shortest possible travel time? What distance $x$ will result in the shortest travel time?

Find minimum of $T(x)$ on calculator.

Minimum value of $T(x) = \text{minimum travel time} = 1.73 \text{ hrs}$

Minimum occurs when $x = 4.50 \text{ miles}$
12. A very large bottle contains 2000 mL of 10% acid solution. An 80% acid solution is being poured into the bottle at a rate of 10 mL/sec.

(a) Express the concentration $C$ of the bottle as a function of time $t$.

\[
\begin{align*}
\frac{10\%}{2000 \text{ mL}} + \frac{80\%}{10t \text{ mL}} &= C \left( \frac{2000 + 10t}{2000 + 10t} \right) \\
\text{Acids:} & \quad 0.1(2000) + 0.8(10t) = C(2000 + 10t) \\
200 + 8t &= C(2000 + 10t) \\
\frac{200 + 8t}{2000 + 10t} &= C(t) \\
(C \text{ is concentration as a decimal})
\end{align*}
\]

(b) When will the concentration be 60%?

For what $t$ does $C = 0.60$?

\[
\begin{align*}
\frac{200 + 8t}{2000 + 10t} &= 0.60 \\
200 + 8t &= 0.60(2000 + 10t) \\
200 + 8t &= 1200 + 6t \\
2t &= 1000 \\
t &= 500 \text{ seconds}
\end{align*}
\]
13. Find the average rate of change of the function \( f(x) = \frac{x^2}{x+1} \) from \( x = 5 \) to \( x = 5 + h \).

\[
\frac{f(5+h) - f(5)}{5+h-5} = \frac{(5+h)^2}{5+h+1} - \frac{5^2}{5+1}
\]

\[
= \frac{(5+h)^2 - \frac{25}{6}}{6+h} \cdot \frac{6(6+h)}{6(6+h)}
\]

\[
= \frac{6(5+h)^2 - 25(6+h)}{6h(6+h)} = \frac{6(25 + 10h + h^2) - 150 - 25h}{6h(6+h)}
\]

\[
= \frac{150 + 60h + 6h^2 - 150 - 25h}{6h(6+h)} = \frac{6h^2 + 35h}{6h(6+h)}
\]

\[
= \frac{6h + 35}{6h(6+h)} = \frac{6h + 35}{6(6+h)}
\]
14. Solve the following equations.

(a) \(3x^{1/3} + 2x^{-2/3} - 2x^{-5/3} = 0\)

\[x^{-5/3} (3x^2 + 2x - 2) = 0\]

\[x^{-5/3} = 0\]

\[x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-2)}}{2(3)}\]

\[= \frac{-2 \pm \sqrt{4 + 24}}{6} = \frac{-2 \pm \sqrt{28}}{6}\]

\[= \frac{-2 \pm 2\sqrt{7}}{6} = \frac{-1 \pm \sqrt{7}}{3}\]
\[(x-2)(x+2)(3x+1)\]

\[\frac{1}{x-2} + \frac{11}{3x+1} = \frac{28}{(x-2)(x+2)(3x+1)}\]

\[(x-2)(x+2)\]

\[(x+2)(3x+1) + 11(x-2)(x+2) = 28\]

\[3x^2 + x + 6x + 2 + 11(x^2 - 4) = 28\]

\[3x^2 + 7x + 2 + 11x^2 - 44 = 28\]

\[14x^2 + 7x - 70 = 0\]

\[7(2x^2 + x - 10) = 0\]

\[7(2x + 5)(x - 2) = 0\]

\[x = -\frac{5}{2}, 2\]

\[x = 2\] makes denominator 0 in original equation.

\[x = 2\] is extraneous.

\[x = -\frac{5}{2}\]
(c) \( 9x^3 - 18x^2 - 4x + 8 = 0 \)

\[
(9x^3-18x^2) + (-4x + 8) = 0
\]

\[
9x^2(x-2) - 4(x-2) = 0
\]

\[
(x-2)(9x^2-4) = 0
\]

\[
(x-2)(3x-2)(3x+2) = 0
\]

\[
x = 2, \quad \frac{2}{3}, \quad -\frac{2}{3}
\]
(d) \( \sqrt{4x-19} + 4 = x \)

\[
(\sqrt{4x-19})^2 = (x - 4)^2
\]

\[4x - 19 = x^2 - 8x + 16\]

\[0 = x^2 - 12x + 35\]

\[0 = (x - 5)(x - 7)\]

\[x = 5, 7\]

Check: \( x = 5: \sqrt{20 - 19} + 4 = 5 \checkmark \)

\[x = 7: \sqrt{28 - 19} + 4 = 7 \checkmark \]

\[x = 5, 7\]
(e) \(16x - 24\sqrt{x} + 9 = 0\)

\[
16(\sqrt{x})^2 - 24(\sqrt{x}) + 9 = 0
\]

Let \(w = \sqrt{x}\)

\[
16w^2 - 24w + 9 = 0
\]

\[
(4w - 3)^2 = 0
\]

\[
w = \frac{3}{4}
\]

\[
\sqrt{x} = \frac{3}{4}
\]

\[
x = \frac{9}{16}
\]

Check:

\[
16\left(\frac{9}{16}\right) - 24\sqrt{\frac{9}{16}} + 9
\]

\[
= 9 - 24\left(\frac{3}{4}\right) + 9
\]

\[
= 9 - 18 + 9 = 0
\]
15. Solve the following inequalities.

(a) \( \frac{4}{2x + 3} \geq 1 \)

\[
\frac{4}{2x + 3} - 1 \geq 0
\]

\[
\frac{4 - (2x + 3)}{2x + 3} \geq 0
\]

\[
\frac{4 - 2x - 3}{2x + 3} \geq 0
\]

\[
\frac{1 - 2x}{2x + 3} \geq 0
\]
(b) $-|6x - 11| + 5 \leq 3$

$-|6x - 11| \leq -2$

$|6x - 11| \geq 2$

$6x - 11 \leq -2$

$6x \leq 9$

$x \leq \frac{9}{6}$

$x \leq \frac{3}{2}$

OR

$6x - 11 \geq 2$

$6x \geq 13$

$x \geq \frac{13}{6}$

$(-\infty, \frac{3}{2}] \cup [\frac{13}{6}, \infty)$
16. Simplify the following expression and write without negative exponents:

\[
\left( \frac{25x^{4/2} y^{-2}}{z^6} \right)^{3/2} \left( \frac{y^{-3} z}{x^3} \right)^{-3}
\]

\[
= \left( \frac{25^{3/2} x^{6} y^{-3}}{z^9} \right) \left( \frac{y^{-9} x^{15}}{z^3} \right)
\]

\[
= \frac{25^{3/2} x^{21} y^{-6}}{z^{12}}
\]

\[
25^{3/2} = (\sqrt{25})^3 = 5^3 = 125
\]
17. Simplify the following expression: \( \sqrt[6]{3^{15}x^{22}y^{14}} \)

\[
\begin{align*}
\sqrt[6]{3^{15}x^{22}y^{14}} &= \sqrt[6]{3^{12} \cdot 3^3 \cdot x^4 \cdot y^2} \\
&= \sqrt[6]{(3^2)^6 \cdot 3^3 \cdot (x^3)^6 \cdot (y^2)^6} \\
&= |3^2 \cdot x^3 \cdot y^2| \cdot \sqrt[6]{3^3 \cdot y^2} \\
&= |3^2| \cdot |x^3| \cdot |y^2| \cdot \sqrt[6]{27 \cdot y^2} \\
&= 9 \cdot |x^3| \cdot |y^2| \cdot \sqrt[6]{27 \cdot y^2} \\
\end{align*}
\]

(Need absolute value because we're taking an even root.)

Since \( y^2 \) is always positive, \( |y^2| = y^2 \).

OR

\[
9 \cdot |x^3| \cdot y^2 \cdot \sqrt[6]{27 \cdot y^2}
\]
18. Find the center and radius of the circle $x^2 + y^2 + 8x - 10y + 37 = 0$.

Complete the square for both $x$ and $y$ terms:

$$\left(x^2 + 8x + 16\right) + \left(y^2 - 10y + 25\right) = -37 + 16 + 25$$

$$= \left(\frac{8}{2}\right)^2$$

$$= \left(\frac{10}{2}\right)^2$$

$$\left(x + 4\right)^2 + \left(y - 5\right)^2 = 4$$

Standard form: $\left(x-h\right)^2 + \left(y-k\right)^2 = r^2$

Center: $(h,k)$; Radius $r$.

Center: $(-4, 5)$; Radius = 2
19. Consider the points (4, 6) and (−6, 2).

(a) Find the distance between these points.

\[
\sqrt{(2-6)^2 + (-6-4)^2} = \sqrt{16 + 100} = \sqrt{116} = \sqrt{4 \cdot 29} = 2\sqrt{29}
\]

(b) Find the equation of the line that is parallel to the line 5x + 8y = 12 and passes through the midpoint of the line segment between these points.

\[
5x + 8y = 12
\]
\[
8y = -5x + 12
\]
\[
y = -\frac{5}{8}x + \frac{3}{2}
\]
Slope is \(-\frac{5}{8}\). Nonvertical parallel lines have equal slope. So the slope of the line we want is
\[
m = -\frac{5}{8}
\]
Midpoint: \(\left(\frac{4 + (-6)}{2}, \frac{6 + 2}{2}\right)\)
\[
= (-1, 4)
\]
Point-Slope Form:
\[
y - y_1 = m(x - x_1)
\]
\[
y - 4 = -\frac{5}{8}(x + 1)
\]
\[
y - 4 = -\frac{5}{8}x - \frac{5}{8}
\]
\[
y = -\frac{5}{8}x - \frac{5}{8} + 4
\]
\[
y = -\frac{5}{8}x - \frac{5}{8} + \frac{32}{8}
\]
\[
y = -\frac{5}{8}x + \frac{27}{8}
\]
20. Consider the function \( f(x) = \sqrt{|x - 2|} + x^4 - 5x^2 + 2x + 3 \).

(a) Find the \( x \)-intercepts of \( f \).

Graph on Calculator.

Use "zero" command to find \( x \)-intercepts:

\((-2.2066, 0) \) and \((-0.8368, 0)\)

(b) Where is \( f \) decreasing? \( \rightarrow \) function goes down from left to right.

Use "maximum" and "minimum" commands to find points.

\( f \) is decreasing on the intervals:

\((-\infty, -1.6618) \cup (0.1649, 1.5125)\)

(c) What is the range of \( f \)?

\([-4.5916, \infty)\)

\( y \)-values that \( f(x) \) achieves.

(d) Solve the equation \( f(x) = x^{1/3} - 2 \).

Leave \( y_1 \) as \( f(x) \)

Set \( y_2 = x^{1/3} - 2 \)

Use "intersect" command to find where the functions are equal \( \Rightarrow \) where they intersect.

\( x = -1.9739, -1.2515 \)
21. True or False

(a) **TRUE** **FALSE** To rationalize the numerator of \( \frac{\sqrt{x} + 10}{x^2} \), multiply numerator and denominator by \( \sqrt{x} + 10 \).

(b) **TRUE** **FALSE** If \( L_1 \) has slope \(-4\) and \( L_2 \) is perpendicular to \( L_1 \), then \( L_2 \) has slope \( 4 \).

(c) **TRUE** **FALSE** The equation \( y = -\frac{1}{20} x^2 + x - 5 \) has exactly one real solution.

\[ D = b^2 - 4ac = 1 - 4 \left( -\frac{1}{20} \right)(-5) = 1 - 1 = 0 \]

(Discriminant) **If** \( D = 0 \) **then** exactly 1 real solution.

(d) **TRUE** **FALSE** A graph that is symmetric about the \( x \)-axis and the \( y \)-axis must also be symmetric about the origin.

Diagram:

```
(x, y)...

(\(\pm\)x, y)...

(\(\pm\)x, \(\pm\)y)...
```