1. Find the equation of the line that

(a) passes through the $y$-intercept of the line $-2x + 3y = 9$ and is parallel to the line $7x - 4y = 6$.

- $y$-int of $-2x + 3y = 9$
  - Set $x=0$: $-2(0) + 3y = 9$ OR $-2x + 3y = 9$  
  - $3y = 9$  
  - $y = 3$

- $y$-int is $(0, 3)$  
  - Parallel $\Rightarrow$ same slope. $7x - 4y = 6$  
  - $-4y = -7x + 6$  
  - $y = \frac{7}{4}x - \frac{3}{2}$  
  - $m = \frac{7}{4}$

- $y = mx + b$; $y = \frac{7}{4}x + 3$

(b) passes through the $x$-intercept of the line $3x - 8y = 12$ and is perpendicular to the line $x = 3$.

- $x$-int: Set $y=0$
  - $3x - 8(0) = 12$
  - $3x = 12$
  - $x = 4$
  - $(4, 0)$

- $y = 0$ (the $x$-axis)

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2. Suppose that the relationship between the cost of utilities and the average temperature in a month is linear. If the average temperature in a month is 96°, your utilities bill is $100. If the average temperature in a month is 81°, your utilities bill is $75.

(a) Find an equation that expresses the cost of your utilities, C, in terms of the average temperature, T, in any given month.

\[ (x_1, y_1) \rightarrow (T, C) \]

\[ (96, 100) \]
\[ (81, 75) \]

\[ m = \frac{\Delta C}{\Delta T} = \frac{100 - 75}{96 - 81} = \frac{25}{15} = \frac{5}{3} \]

\[ y - y_1 = m(x - x_1) \]
\[ C - C_1 = m(T - T_1) \]
\[ C - 100 = \frac{5}{3}(T - 96) \]
\[ C - 100 = \frac{5}{3}T - 160 \]
\[ C = \frac{5}{3}T - 60 \]

(b) How much will your utilities bill increase if the average temperature in the current month is 6° higher than the average temperature last month?

\[ m = \frac{\Delta C}{\Delta T} = \frac{5}{3} \]

Given that \( \Delta T = 6° \), \( \Delta C = \frac{5}{3} \cdot 6 = 10 \)

\[ $10 higher \]
3. Find an equation of the perpendicular bisector of the line segment joining the points \((-1, 2)\) and \((4, 3)\).

Midpoint: \(\left(\frac{-1+4}{2}, \frac{2+3}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}\right)\)

\[m = \frac{\frac{3}{2} - 2}{4 - (-1)} = \frac{1}{5}\]  
(of line segment)

\[m_\perp = -5 \quad \text{(slope of perpendicular bisector)}\]

\[y - y_1 = m(x - x_1)\]

\[y - \frac{5}{2} = -5\left(x - \frac{3}{2}\right)\]

\[y - \frac{5}{2} = -5x + \frac{15}{2}\]

\[y = -5x + 10\]
4. Determine whether the following equations define $y$ as a function of $x$.

(a) $x^2 + y^2 = 16$

\[
y^2 = 16 - x^2
\]

\[
y = \pm \sqrt{16 - x^2}
\]

NOT a function

(b) $x^3y + 4y = 12$

\[
y(x^3 + 4) = 12
\]

\[
y = \frac{12}{x^3 + 4}
\]

YES, is a function

(c) $y^3 - x = 1$

\[
y^3 = 1 + x
\]

\[
y = \sqrt[3]{1 + x}
\]

YES, is a function
5. Find the domains of the following functions.

(a) \( f(x) = \frac{x^3}{\sqrt{x^2 - 9}} \)

\[ x^2 - 9 > 0 \]
\[ (x - 3)(x + 3) > 0 \]

\((-\infty, -3) \cup (3, \infty)\)
(b) \( f(x) = \frac{\sqrt{x^2 - 6x - 16}}{x^2 + 4x - 21} \).

\[ x^2 + 4x - 21 \neq 0 \]
\[ (x + 7)(x - 3) = 0 \]
\[ x \neq -7, 3 \]

\[ x^2 - 6x - 16 \geq 0 \]
\[ (x - 8)(x + 2) \geq 0 \]

\[ \begin{array}{cccc}
- & -2 & + & 8 & + \\
\hline
- & + & - & +
\end{array} \]

\[ (-\infty, -7) \cup (-7, -2] \cup [8, \infty) \]
6. Let \( f(x) = \frac{x^2 + 1}{2 - x} \). Evaluate the following.

(a) \( f\left(\frac{1}{x}\right) = \frac{\left(\frac{1}{x}\right)^2 + 1}{2 - \left(\frac{1}{x}\right)} = \frac{\frac{1}{x^2} + 1}{2 - \frac{1}{x}} = \frac{1 + x^2}{x^2} \cdot \frac{x}{2x-1} = \frac{1+x^2}{x(2x-1)} \)

(b) \( f(-x^2) = \frac{(-x^2)^2 + 1}{2 - (-x^2)} = \frac{x^4 + 1}{2 + x^2} \)
7. Consider the function:

\[ f(x) = \begin{cases} 
-\frac{1}{2}x + 2 & \text{if } x \leq -1 \\
-x^2 & \text{if } -1 < x \leq 1 \\
3 & \text{if } 1 < x < 4
\end{cases} \]

(a) Graph the function.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>(-\frac{1}{2}(-2)+2 = 3)</td>
</tr>
<tr>
<td>-1</td>
<td>(-\frac{1}{2}(-1)+2 = \frac{5}{2})</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
(b) What are the domain and range of $f$?

Domain: $(-\infty, 4)$
Range: $[0, 1] \cup [\frac{5}{2}, \infty)$

(c) On what intervals is $f$ increasing? decreasing?

Increasing on $(0, 1)$
Decreasing on $(-\infty, -1) \cup (-1, 0)$
8. Graph the function \( f(x) = |x^2 - 4| \) by plotting points.

\[
\begin{array}{c|c}
 x & f(x) \\
-3 & |(-3)^2 - 4| = 5 \\
-2 & |(-2)^2 - 4| = 0 \\
-1 & |(-1)^2 - 4| = 3 \\
 0 & |0^2 - 4| = 4 \\
 1 & |1^2 - 4| = 3 \\
 2 & |2^2 - 4| = 0 \\
 3 & |3^2 - 4| = 5 \\
\end{array}
\]

If it were just \( f(x) = x^2 - 4 \).
9. Graph the function \( f(x) = x^4 - 5x^3 - 3x^2 + 17x - 10 \) using a graphing calculator.

(a) What is the range of this function? (Round decimals to 4 places.)

\[ [-54.6444, \infty) \]

(b) On what intervals is \( f \) increasing? decreasing? (Round decimals to 4 places.)

Increasing on:
\[ (-1.1030, 1) \cup (3.8530, \infty) \]

Decreasing on:
\[ (-\infty, -1.1030) \cup (1, 3.8530) \]
Average rate of change of \( f(x) \) from \( x=a \) to \( x=b \):

\[
\frac{f(b) - f(a)}{b - a}
\]
10. Find the average rate of change for the following functions on the given interval.

(a) \( f(x) = \sqrt{x + 8} \) from \( x = -4 \) to \( x = 1 \)

\[
\frac{f(1) - f(-4)}{1 - (-4)} = \frac{\sqrt{1+8} - \sqrt{-4+8}}{5} = \frac{3 - 2}{5} = \frac{1}{5}
\]
(b) \( f(x) = x^2 + 2x - 4 \) from \( x = 2 \) to \( x = 2 + h \)

\[
\frac{f(2+h) - f(2)}{2+h - 2} = \frac{[(2+h)^2 + 2(2+h) - 4] - [2^2 + 4 - 4]}{h}
\]

\[
= \frac{4 + 4h + h^2 + 4 + 2h - 4 - 4}{h}
\]

\[
= \frac{6h + h^2}{h} = \frac{h(6+h)}{h}
\]

\[
= 6 + h
\]
(c) \( f(x) = \frac{5}{x-4} \) from \( x = a \) to \( x = a + h \)

\[
\frac{f(a+h) - f(a)}{a+h - a} = \frac{\frac{5}{a+h-4} - \frac{5}{a-4}}{h}
\]

\[
= \frac{5(a-4) - 5(a+h-4)}{(a+h-4)(a-4)}
\]

\[
= \frac{5a - 20 - 5a - 5h + 20}{(a+h-4)(a-4)}
\]

\[
= \frac{-5h}{h}
\]

\[
= \frac{-5}{(a+h-4)(a-4)}
\]
11. Suppose an object is launched into motion. After 10 seconds, the object has traveled 220 feet. After 15 seconds, the object has traveled a total of 450 feet.

(a) What was the object’s average speed during the first 10 seconds?

\[
\text{average speed} = \frac{\text{distance traveled}}{\text{time}} = \frac{220}{10} = 22 \text{ ft/sec}
\]

\[
\frac{d(10) - d(0)}{10 - 0} = \frac{220 - 0}{10 - 0} = \frac{220}{10} = 
\]

(b) What was the car’s average speed during the last 5 seconds?

\[
\text{average speed} = \frac{\text{distance traveled}}{\text{time}} = \frac{450 - 220}{5} = \frac{230}{5} = 46 \text{ ft/sec}
\]

\[
\frac{d(15) - d(10)}{15 - 10} = \frac{450 - 220}{5} 
\]
12. If the distance in feet an object has traveled after \( t \) seconds is modeled by the function \( f(t) = t^3 + 6t \), then what is the object's average speed from \( t = a \) to \( t = a + h \)?

\[
\text{average speed} = \text{average rate of change of } f \\
\frac{f(a+h) - f(a)}{a+h - a} = \frac{[(a+h)^3 + 6(a+h)] - [a^3 + 6a]}{h}
\]

\[
= \frac{a^3 + 3a^2h + 3ah^2 + h^3 + 6a + 6h - a^3 - 6a}{h}
\]

\[
= \frac{3a^2h + 3ah^2 + h^3 + 6h}{h}
\]

\[
= h(3a^2 + 3ah + h^2 + 6)
\]

\[
= \frac{3a^2 + 3ah + h^2 + 6}{f + 1 \text{ sec}}
\]