9.1 Systems of Equations

A **system of equations** is a set of equations that all involve the same variables. A solution of a system is an assignment of values to each variable that makes EVERY equation in the system true.

There are 3 methods of solving systems of equations: Substitution, Elimination, and Graphing.

**Substitution Method**

1. Solve for one variable in one of the equations.
2. Substitute this expression into the other equation and solve for that variable.

Example: Solve this system of equations using substitution.
\[
\begin{align*}
  x^2 + y &= 9 \\
  x - y + 3 &= 0
\end{align*}
\]

**Elimination Method**

1. Adjust the coefficients: Multiply one or more of the equations by number(s) that will make the coefficient of a term in one equation the negative of the coefficient in the other equation.
2. Add the equations and solve.

Example: (9.1 #6) Solve this system of equations using the method of elimination.
\[
\begin{align*}
  2x - 2y &= -6 \\
  6x^2 + 3y^2 &= 12
\end{align*}
\]
Example: Solve the following system of equations.
\begin{align*}
xy &= 12 \\
2x^2 - y^2 - 2 &= 0
\end{align*}

Graphical Method

1. Solve each equation for \( y \) so that you can put them in your calculator.
2. Graph the equations.
3. Find the points of intersection.

Example: (9.1 #41) Solve the following system graphically. Round solutions to 2 decimal places.
\begin{align*}
x^2 + y^2 &= 25 \\
x^2 - y &= 2x + 2
\end{align*}

10.1-10.3 Parabolas, Ellipses, and Hyperbolas

We’ve looked at parabolas before when talking about the graphs of quadratic functions.

Recall that the standard form of a parabola that opens up or down with vertex \((h, k)\) is: \( y = a(x - h)^2 + k \).
If \( a > 0 \), the parabola opens upward. If \( a < 0 \), the parabola opens downward.

We can also have parabolas that open left or right, even though these will not be functions.

The standard form a parabola that opens left or right with vertex \((h, k)\) is \( x = a(y - k)^2 + h \).
If \( a > 0 \), the parabola opens to the right. If \( a < 0 \), the parabola opens to the left.
Examples: Sketch the general shape of the following equations.

1. \( y = -2(x + 3)^2 + 4 \)

2. \( y^2 + 10y = -6x - 13 \)

An ellipse is the set of all points where the sum of the distances from two fixed points \( F_1 \) and \( F_2 \) is a constant. These two fixed points are called the foci (plural of focus) of the ellipse.

An ellipse is essentially a circle that has been stretched or shrunk horizontally and/or vertically.

An ellipse will be more elongated vertically or horizontally. An ellipse has a vertical major axis if it is more elongated vertically and a horizontal major axis if it is more elongated horizontally.

The equation of an ellipse with center \((h, k)\) is: \( \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1 \)

If \( a > b \), the major axis is horizontal. If \( a < b \), the major axis is vertical.
Examples: Sketch graphs for the following equations.

1. \(4x^2 + 25y^2 = 100\)

2. \(\frac{(x - 2)^2}{25} + \frac{(y + 3)^2}{4} = 1\)

3. \(9x^2 - 36x + 4(y - 6)^2 = 0\)
A hyperbola is the set of all points where the difference of the distances from two fixed points $F_1$ and $F_2$ is a constant. These two fixed points are called the foci of the hyperbola.

A hyperbola consists of two branches. The segment joining the two branches of the hyperbola is called the transverse axis. The transverse axis can be vertical or horizontal.

The equation of a hyperbola with center $(h, k)$ and horizontal transverse axis is: 
\[ \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \]

The equation of a hyperbola with center $(h, k)$ and vertical transverse axis is: 
\[ \frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1 \]

Examples: Sketch graphs for the following equations.

1. $9x^2 - 4y^2 = 36$

2. $9(x + 2)^2 - 4(y - 3)^2 = 36$.

3. $5y^2 + 20y - 6x^2 + 50 = 0$