Chapter 5, continued

5.3 Trigonometric Graphs

Definition: A function $f$ is called periodic if it repeats itself over and over. Mathematically, there is a positive number $p$ such that $f(t + p) = f(t)$ for every $t$.

The smallest length $p$ that is repeated is called the period of $f$.

Then, every interval of length $p$ of the graph of $f$ is one complete period of $f$. The graph repeats itself every interval of length $p$.

Both sine and cosine are periodic functions with period $2\pi$.

\[
\sin(t + 2\pi) = \sin t \\
\cos(t + 2\pi) = \cos t
\]

Sketch the graph of $f(t) = \sin t$.

Sketch the graph of $f(t) = \cos t$. 

1
Once again, we can use transformations on these graphs to find other trig graphs. In general, it is only necessary to graph one complete period.

Sketch the graph of \( f(t) = -2 \sin t + 1 \).

The **amplitude** of a sine or cosine function is the height of the curve, measured from the center of the graph.

The sine and cosine curves

\[
y = a \sin kx \quad \text{and} \quad y = a \cos kx \quad (k > 0)
\]

have amplitude \(|a|\) and period \(\frac{2\pi}{k}\).

Remember that if \(0 < k < 1\), then the graph will be stretched horizontally, and if \(k > 1\), then the graph will be shrunk horizontally.

Find the amplitude and period of the functions below and graph one complete period.

\( y = 4 \sin 2x \)

\( y = 5 \cos \frac{1}{4}x \)
The sine and cosine curves

\[ y = a \sin k(x - b) \quad \text{and} \quad y = a \cos k(x - b) \quad (k > 0) \]

have amplitude \(|a|\), period \(\frac{2\pi}{k}\), and phase shift \(b\).

Find the amplitude, period, and phase shift of the functions below and describe how they would be graphed.

\[ y = 2 \sin 3(x - \frac{\pi}{3}) \]

\[ y = 3 \cos(4x + \pi) \]

5.4 More Trigonometric Graphs

The functions tangent and cotangent have period \(\pi\).

\[ \tan(x + \pi) = \tan x \quad \text{and} \quad \cot(x + \pi) = \cot x \]

Recall that tangent is not defined at \(\frac{\pi}{2}\) or at \(-\frac{\pi}{2}\), (or \(\frac{\pi}{2} + n\pi\) for any integer \(n\)). Thus the graph of tangent has vertical asymptotes at these values.

\[ \tan x \rightarrow \infty \quad \text{as} \quad x \rightarrow \frac{\pi}{2}^- \]

\[ \tan x \rightarrow -\infty \quad \text{as} \quad x \rightarrow \frac{\pi}{2}^+ \]

Sketch the graph of \(y = \tan x\).
Cotangent is not defined at $n\pi$ for any integer $n$. The graph of cotangent has vertical asymptotes at these values.

$$\cot x \to \infty \text{ as } x \to 0^+$$

$$\cot x \to -\infty \text{ as } x \to \pi^-$$

Sketch the graph of $y = \cot x$.

The functions $y = a \tan kx$ and $y = a \cot kx$ ($k > 0$) have period $\frac{\pi}{k}$.

Find the periods of the following functions and describe how the function would be graphed.

$y = \tan\left(\frac{1}{2}x + \frac{\pi}{8}\right)$

$y = \cot 2\left(x - \frac{3\pi}{4}\right)$
The functions cosecant and secant have period $2\pi$.

$$\csc(x + 2\pi) = \csc x \text{ and } \sec(x + 2\pi) = \sec x$$

Use the facts that $\csc x = \frac{1}{\sin x}$ and $\sec x = \frac{1}{\cos x}$ to sketch the graphs of $y = \csc x$ and $y = \sec x$.

The functions $y = a \csc kx$ and $y = a \sec kx$ ($k > 0$) have period $\frac{2\pi}{k}$ (since they are reciprocals of sine and cosine).

Find the periods of the following and describe how the function would be graphed.

$y = 5 \sec 2\pi x$

$y = \frac{1}{2} \csc(2x - \frac{\pi}{3})$