5.1 Simple and Compound Interest

If $P$ dollars (called the principal or present value) earns interest at a simple interest rate of $r$ per year (as a decimal) for $t$ years, then:

**Interest:** $I = Prt$

**Future Value or Accumulated Amount:** $F = P + I = P + Prt = P(1 + rt)$

Examples:

1. How much interest is earned on a $1000 investment made for 3 yr at a simple interest rate of 5%/year. What is the accumulated amount?

2. A bank deposit paying simple interest at the rate of 8%/yr grew to a sum of $2650 in 9 months. What was the principal?

3. You take out a loan for $3000 that is accruing simple interest. After 5 months, you owe $3112.50.
   
   (a) What is the simple interest rate being charged on this loan?

   (b) After how many months from the beginning of the loan will you owe $3450?
Suppose a principal $P$ earns interest at an annual interest rate of $r$ and interest is compounded $m$ times a year. Then, after $t$ years, the accumulated amount or future value $F$ is:

$$F = P \left(1 + \frac{r}{m}\right)^{mt}$$

Compound interest is different from simple interest in that each time interest is calculated, the accumulated amount becomes the new principal. In other words, the interest earns interest. Instead of using this formula, we will use the TVM Solver in your calculator. However, the TVM Solver CANNOT be used for simple interest calculations...only compound interest.

To get to the TVM Solver, press APPS, select 1:Finance and select 1:TVM Solver.

(Note: If you have a regular TI-83, (not Plus), then press 2nd $x^{-1}$, which is the FINANCE button.)

- $N$ = the total number of compounding periods. ($N = mt$)
- $I\%$ = the interest rate as a percentage.
- $PV$ = present value/principal.
- $PMT$ = regular payment amount per period. (0 if there are no regular payments.)
- $FV$ = future value/accumulated amount.
- $P/Y = C/Y$ = number of compounding periods per year. (This is the same thing as $m$.)
- $PMT$: END BEGIN (For this class, always have END selected.)

To solve for the value you want, move the cursor to that position and then press ALPHA, ENTER.

Important note: In the TVM Solver, the values for PV, PMT, and FV will sometimes be negative. This is done to represent the transfer or flow of money. We will usually look at these problems from the standpoint of the investor or borrower.

A negative number represents an outflow of money away from the investor or borrower, i.e. when money is leaving your pocket. Use a negative number when:

- Making payments
- Depositing money in a bank

A positive number represents an inflow of money to the investor or borrower, i.e. when you put money in your pocket. Use a positive number when:

- You receive a loan from a bank or lender.
- You receive money from a bank account.
Examples:

1. Suppose I invest $1000 in an account at an annual interest rate of 9% per year. After 5 years, how much is in my account if interest is:

   (a) compounded annually? How much interest is earned?
   \[ N = \text{PMT} \]
   \[ I\% = \text{FV} \]
   \[ PV = \text{P/Y=C/Y} \]

   (b) compounded semiannually? How much interest is earned?
   \[ N = \text{PMT} \]
   \[ I\% = \text{FV} \]
   \[ PV = \text{P/Y=C/Y} \]

   (c) compounded daily? How much interest is earned?
   \[ N = \text{PMT} \]
   \[ I\% = \text{FV} \]
   \[ PV = \text{P/Y=C/Y} \]

2. If you want to have $40,000 in 4 years, how much should you invest in an account earning interest at 8%/yr compounded weekly?

   \[ N = \text{PMT} \]
   \[ I\% = \text{FV} \]
   \[ PV = \text{P/Y=C/Y} \]

   How much total interest will be earned on your money?

3. How many years will it take for a deposit of $20,000 to grow to $34,000 at an interest rate of 8.5%/yr compounded quarterly?

   \[ N = \text{PMT} \]
   \[ I\% = \text{FV} \]
   \[ PV = \text{P/Y=C/Y} \]

   In order for the account to grow to $34,000 in only 5 years, what would the annual interest rate have to be (rounded to 4 decimal places)?

   \[ N = \text{PMT} \]
   \[ I\% = \text{FV} \]
   \[ PV = \text{P/Y=C/Y} \]
If interest is compounded \textit{continuously} at an interest rate of \( r \) for \( t \) years, we use the formula:

\[ F = Pe^{rt} \]

Example: If I invest $1000 at 9\%/yr with interest compounded continuously, how much is in my account after 10 months?

Suppose I have $P$ invested at an interest rate of \( I\% /\text{year} \) with interest compounded \( m \) times a year. The \textbf{effective interest rate} is the simple interest rate that would yield the same accumulated amount in one year as the above account. The effective interest rate is often used when comparing two accounts that are compounded differently.

The effective interest rate can be found on your calculator by going to the Finance menu and then scrolling down to \( C:Eff \). Then, enter in \( I \) (as a percent) and \( m \).

\[ Eff(I, m) \]

Example: Bank A has a nominal interest rate of 4.25\%/year compounded weekly, Bank B has an interest rate of 4.2\%/year compounded daily. Bank C has an interest rate of 4.22\%/year compounded quarterly. Which bank offers the best rate of interest for investment? Which bank offers the best rate of interest for a loan?

\section*{5.2 - 5.3 Annuities, Sinking Funds, and Amortization}

An \textbf{annuity} is a series of payments made at \textit{regular} time intervals, such as every month, every quarter, every day, etc. With annuities, \textbf{PMT} will no longer just be 0 on the TVM Solver.

Also, remember that for this class, the payments will always be made at the END of each period.

Examples:

1. Suppose I make monthly deposits of $150 into my bank account for 15 years at 3\%/yr compounded monthly?

(a) How much money is in my bank account at the end of the 15 years?

\begin{align*}
& N= \quad \text{PMT}= \\
& I\%= \quad \text{FV}= \\
& PV= \quad \text{P/Y}=\text{C/Y}= \\
\end{align*}

(b) How much total interest did I earn?
2. Suppose a person opens up a retirement account with $2000 and then makes quarterly deposits of $1500. The account earns interest at a rate of 4.7%/yr compounded quarterly.

(a) How much will be in the account when this person retires in 30 years?

\[
\begin{align*}
N &= \text{ } \\
PMT &= \text{ } \\
I\% &= \text{ } \\
FV &= \text{ } \\
PV &= \text{ } \\
P/Y &= \text{ } \\
C/Y &= \text{ } \\
\end{align*}
\]

(b) How much interest is earned in total?

3. A sinking fund is an account that is set up for a specific purpose at some future date.

(a) A family wants to save up some money to make a $40,000 down payment on a house in 7 years. How much should they deposit each month into an account if the account earns interest at the rate of 8.5%/year compounded monthly?

\[
\begin{align*}
N &= \text{ } \\
PMT &= \text{ } \\
I\% &= \text{ } \\
FV &= \text{ } \\
PV &= \text{ } \\
P/Y &= \text{ } \\
C/Y &= \text{ } \\
\end{align*}
\]

(b) How much of their money will come from interest?

(c) If they can afford to put in $400 a month instead, in how many years can they afford the down payment?

\[
\begin{align*}
N &= \text{ } \\
PMT &= \text{ } \\
I\% &= \text{ } \\
FV &= \text{ } \\
PV &= \text{ } \\
P/Y &= \text{ } \\
C/Y &= \text{ } \\
\end{align*}
\]
Sometimes it is necessary to determine how much money is needed in an account now so that regular payments can be made in the future.

Examples:

1. Suppose you win a lottery worth $1,000,000 which is paid out with $40,000 payments for the next 25 years. In order to make these payments to you for the next 25 years, how much money must the lottery commission have in an account now if the account earns interest at a rate of 2.5%/yr compounded annually?

N=  
I%=  
PMT=  
FV=  
PV=  
P/Y=C/Y=  

If you are given the option of taking a lump sum of $750,000 now or of accepting the payments, which should you take?

A common type of annuity is a loan or mortgage. To pay off a debt with regular payments is called amortization. When dealing with loans or mortgages, PV should be the original amount of the loan. The FV of a loan or mortgage should be 0 since you will completely pay back all the money loaned to you.

2. Suppose you are granted a student loan for $10,000. When you graduate, you will be required to make monthly payments for 15 years.

(a) How much will you be required to pay each month if the interest rate on the loan is 6%/yr compounded monthly?

N=  
PMT=  
I%=  
FV=  
PV=  
P/Y=C/Y=  

(b) How much total interest will you pay on the loan?

3. A business purchased a piece of property for $2,000,000. They made an initial down payment of 10% and obtained financing for the balance. If the loan is to be amortized over 15 yr at an interest rate of 12%/year compounded quarterly, find the required quarterly payment.

N=  
PMT=  
I%=  
FV=  
PV=  
P/Y=C/Y=  

6
4. Greg bought a car. He made a down payment of $3000 and financed the remaining balance by taking out a loan. He will have to pay $300 per month for 4 years at 7% interest compounded monthly in order to pay off the car.

(a) What was the original cash price of the car?

N= PMT=
I%= FV=
PV= P/Y=C/Y=

(b) How much interest will he pay by the time the car is paid off?

5. A family secured a 25-year bank loan of $150,000 to purchase a house. The bank charges interest at a rate of 9%/yr compounded monthly.

(a) What is their monthly payment?

N= PMT=
I%= FV=
PV= P/Y=C/Y=

Note: The amount of interest they will pay is:

\[(1258.79)(25)12) − 150,000 = 377,637 − 150,000 = 227,637\]

Outstanding principal is how much you still owe at a given point. To find the outstanding principal, find the present value of the remaining payments.

(b) What is the outstanding principal after 8 years? In other words, how much do they still owe after 8 years?

N= PMT=
I%= FV=
PV= P/Y=C/Y=

(c) How much have they actually paid out of pocket in these 8 years? Why not subtract this from $150,000 to find the outstanding principal?

Equity in a loan scenario is how much of the item you actually own and the bank doesn’t. It is how much principal you have paid on the original loan (what belongs to you). The interest you pay does NOT count towards your equity. At any moment in time, the following is true:

\[\text{Value of Item} = \text{Equity} + \text{Outstanding Principal}\]

(d) What is their equity after 8 years?
6. Four years ago, Emily bought a $200,000 home by making a 20% down payment and financing the remaining balance. The loan was at 7.5%/year compounded monthly and the term of the loan was 30 years.

(a) What is Emily’s current monthly mortgage payment?

\[
\begin{align*}
N &= \text{PMT} = \\
I\% &= \text{FV} = \\
PV &= \text{P/Y=C/Y} = \\
\end{align*}
\]

(b) After these first four years, Emily decides to refinance her home. What is her outstanding principal at this point? Equity?

\[
\begin{align*}
N &= \text{PMT} = \\
I\% &= \text{FV} = \\
PV &= \text{P/Y=C/Y} = \\
\end{align*}
\]

(c) Emily refinanced her home by securing a new 20-year loan for the outstanding principal at a new rate of 6.75%/year compounded monthly. What will be her new monthly mortgage payment now?

\[
\begin{align*}
N &= \text{PMT} = \\
I\% &= \text{FV} = \\
PV &= \text{P/Y=C/Y} = \\
\end{align*}
\]

(d) How much money (interest) will Emily save by refinancing the loan?

Analyzing individual payments:

Every time you make a payment on a loan, part of this payment is going towards the principal and part of the payment is going towards the interest that has accrued during that period. In order to figure out how much of a single payment goes toward interest, we use the interest rate PER PERIOD, (the periodic interest rate). This periodic interest rate is found by dividing the annual interest rate by the number of periods per year.
7. Suppose I take out a 2-year loan in the amount of $5000 at an interest rate of 6.5%/yr compounded quarterly in order to buy a used car.

(a) What is my current quarterly payment?

\[
\begin{align*}
N &= \quad \text{PMT}= \\
I\% &= \quad \text{FV}= \\
PV &= \quad P/Y=C/Y= \\
\end{align*}
\]

(b) How much of the first payment goes toward interest?

(c) How much goes toward the principal?

(d) What is the outstanding principal now? Equity?

(e) How much of the second payment goes toward interest and how much goes toward the principal? What is the outstanding principal now? Equity?

(f) Finish the amortization table for the next 2 periods:

<table>
<thead>
<tr>
<th>Period</th>
<th>Payment</th>
<th>Towards Interest</th>
<th>Towards Principal</th>
<th>Outstanding Principal</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>5,000</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8. A boat costs $65,000. You make a down payment of $10,000 and finance the remaining balance with a 10-year loan at an interest rate of 5.1%/yr compounded monthly. You find that your monthly payment is $586.05. (Check this on your own!)

(a) Fill in the amortization table for the first 2 monthly payments.

<table>
<thead>
<tr>
<th>Period</th>
<th>Payment</th>
<th>Towards Interest</th>
<th>Towards Principal</th>
<th>Outstanding Principal</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>1</td>
<td>586.05</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>586.05</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

(b) What are the outstanding principal and equity after 7 years? (Hint: Don’t analyze all 84 payments that have been made.)

\[
\begin{align*}
N &=  & \text{PMT} &= \\
I\% &=  & \text{FV} &= \\
PV &=  & \text{P/Y}=\text{C/Y} &= \\
\end{align*}
\]