

Greetings. [No, this isn't a notice to report for induction into the armed forces!] It is a list of suggestions for homework problems to do so as to build and maintain skill. Finger-proficiency is important, for it frees up the mind to think about that part of mathematics that requires thought rather than computation. It also is part of building intuition.

p 399 # 88, 90.

p 400 # 92.

p 407 # 51, 64, 78. In 78, as well as solving by interpretation, continue after the first substitution (which you will readily see ought to be  $t = x^2$ ) by substituting  $t = \sin(\theta)$ . This will get you a trigonometric integral which also looks hard, but a trig identity relating squares of sine and cosine to double angles ought to do the trick.

p 407 # 86.

pp 427-428 # 3, 25, 35, 51 [the little icon next to 51 says that you will be needing to use Maple.]

Find the area bounded by the lines from the origin to the curve  $x^2 - y^2 = 1$  at  $(\sqrt{2}, \pm 1)$ .

Now, slightly off topic: One of you asked after class how an exact answer might be found to the question of what values of  $x$  have  $\sin \sin \sin \sin x = \cos \cos \cos \cos x$ . Maple can provide an excellent hint (really, an answer). Just plot the two functions, or plot their difference, over any interval of length  $2\pi$ . It will appear that there is no intersection. But wait! How do we know that the plot is not missing something? Maple plots work by evaluating the function given them at a finite number of values, and then connecting the dots. Could the function take a violent zig up and back down, in between two such dots?

If so, it would have to have a large derivative at some point. So, this gets you thinking about the derivative of  $\sin \sin \sin \sin x$ , say. If you can come up with a reason why this is always small, you will have the theoretical side of the proof to go with the evidence of your eyes.

Finally, how about a simpler proof? Do we really need a hundred dots, and all the arithmetic that went into them? Probably not. Here's some starting ideas:  $\sin x \leq 1$  for all  $x$ . Thus,  $\sin \sin x \leq \sin 1 < \sin(\pi/3) = \sqrt{3}/2$ . And so,  $\sin \sin \sin x < \sin(\sqrt{3}/2)$  and so on. Similarly, there ought to be a series of simple inequalities for  $\cos \cos \cos \cos x$ , but in the other direction.

If these work out, the upper bound for the nested sines will come in less than the lower bound for the nested cosines.