1. Let \( x \) be the number of days Walter does his chores exceptionally well. Then \( 10 - x \) is the number of days he did not do them exceptionally well. He received a total of \( 36 = 5x + 3(10 - x) = 30 + 2x \) dollars, hence \( x = 3 \).

2. Let \( u \) be the man’s usual speed, and let \( v \) be the rate of the stream’s current. Then \( 15/(v + u) = 15/(v - u) - 5 \) and \( 15/(2v + u) = 15/(2v - u) - 1 \), which is equivalent to

\[
\begin{align*}
15(v - u) &= 15(v + u) - 5(v^2 - u^2) \\
15(2v - u) &= 15(2v + u) - (4v^2 - u^2)
\end{align*}
\]

Multiply the first equality by 4 and subtract it from the second:

\[
6u = v^2 - u^2
\]

\[
30u = 4v^2 - u^2
\]

hence \( u = 2 \) (or \( u = 0 \)). Then from the first equality, we get \( 16 = v^2 \), i.e., \( v = 4 \). (If \( u = 0 \), then \( v = 0 \), which is absurd.)

3. Let \( a_n \) be the number of unit squares in figure \( n \). Then \( a_n - a_{n-1} = 4n \), hence \( a_n = 1 + 4 + 8 + \cdots + 4n = 1 + 4(1 + 2 + \cdots + n) = 1 + 2n(n + 1) = 2n^2 + 2n + 1 \). It follows that \( a_{100} = 20000 + 200 + 1 = 20201 \).

4. Let \( a \) be the length of the side of the large square, and let \( x \) be the width of the rectangles. Then perimeter of each of the rectangles is \( 2x + 2(a - x) = 2a = 14 \), hence \( a = 7 \), so that the area of the large square is 49.

5. The system is equivalent to

\[
\begin{align*}
2^x - x - y &= 2^3 \\
3^x + 2^y - 5y &= 3^5
\end{align*}
\]

or to

\[
\begin{align*}
x - y &= 3 \\
2x - 3y &= 5
\end{align*}
\]

Hence the solution is \( x = 4, y = 1 \).

6. \( 51 + 61 + \cdots + 391 = 35(51 + 391)/2 = 35 \cdot 221 = 7735 \)

7. It is equal to \(-1)^{100} + 1 = 2\).

8. If they have a common solution, then \( x^2 + y^2 - 16 = x^2 - 3y + 12 \), hence \( y^2 - 16 = -3y + 12 \), so that \( y^2 + 3y - 28 = 0 \). Solutions of this equation are \( y = 4 \) and \( y = -7 \). Then the first equation is either \( x^2 = 16 - y^2 = 16 - 16 = 0 \), or \( x^2 = 16 - 49 = -33 \). There is no real solution in the second case, hence the only value of \( y \) for which there is a real solution is \( y = 4 \).

9. Note that \( \sin x = \sin 2\pi/5 = \sin 32\pi/5 = \sin 16x \), \( 32\pi/5 = 6\pi + 2\pi/5 \). Multiply the expression by \( 16 \sin x \):

\[
(16 \sin x \cos x \cdot \cos 2x \cos 4x \cos 8x) = (8 \sin 2x \cos 2x \cdot \cos 4x \cos 8x) = (4 \sin 4x \cos 4x \cdot \cos 8x) = 2 \sin 8x \cos 8x = \sin 16x.
\]

It follows that our expression is equal to \( \frac{1}{16} \).

10. Consider \( \triangle BOA \) and \( \triangle COD \). We have \( \angle BOA = \angle COD \), and \( BO : OC = 4 : 3 = 8 : 6 = AO : OD \). It follows that they are similar, and that \( AB : CD = 4 : 3 \). Hence \( CD = 18/4 = 9/2 \).

11. We have \( \sin^2 2x = 4 \sin^2 x \cos^2 x = 4 \sin^2 x(1 - \sin^2 x) \). Denote \( y = \sin^2 x \). Then the original equation becomes \( 2y + 4y(1 - y) = 2 \), or \( 2y^2 - 3y + 1 = 0 \). Its solutions are \( y = 1 \) and \( y = 1/2 \). It follows that \( \sin x = \pm 1 \) or \( \sin x = \pm 1/\sqrt{2} \). Hence, solutions of the equation are \( \pi/4, \pi/2, 3\pi/4, 5\pi/4, 3\pi/2, 7\pi/4 \).
12. Our number is equal to $1100A + 11B = 11(100A + B) = 11(99A + A + B)$. Since it is a perfect square, and divisible by 11, $A + B$ is divisible by 11. But this means that $A + B = 11$, and our number is equal to $11(99A + 11) = 121(9A + 1)$. It follows that $9A + 1$ is a perfect square. Checking all 9 possibilities for $A$, we see that the only case when $9A + 1$ is a perfect square is $A = 7$. Then $B = 4$, and our number is 7744.

13. Since $BQ$ is $42^\circ$, $\angle BAQ = 21^\circ$; and since $QD = 38^\circ$, $\angle DCQ = 19^\circ$. It follows that $\angle PAQ = 159^\circ$, $\angle PCQ = 161^\circ$. Then $\angle APC + \angle AQC = 360^\circ - 159^\circ - 161^\circ = 40^\circ$.

14. Solve the system
\[
\begin{align*}
(x^3 + y^3)(x^2 + y^2) &= 2 \\
x + y &= 1
\end{align*}
\]
We have $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$, hence we can replace the first equation by $(x^2 - xy + y^2)(x^2 + y^2) = 2$.
Denote $x^2 + y^2 = s$, $xy = p$. Then the second equation implies $s + 2p = 1$, and the first equation can be written $(s - p)s = 2$. Replacing $s$ by $1 - 2p$ in the second equation, we get $(1 - 3p)(1 - 2p) = 2$, or $6p^2 - 5p - 1 = 0$. Solutions of this equation are $p = 1$ and $p = -1/6$. The corresponding values of $s$ are $-1$ and $1 + 1/3 = 4/3$. The first case is impossible, hence we get
\[
\begin{align*}
xy &= -1/6 \\
x + y &= 1
\end{align*}
\]
It follows that $x$ and $y$ are roots of the polynomial $t^2 - t - 1/6$. Therefore, they are equal to
\[
\frac{1 + \sqrt{1 + 2/3}}{2} = \frac{1}{2} + \frac{\sqrt{15}}{6}, \quad \frac{1}{2} - \frac{\sqrt{15}}{6}
\]
in some order.

Note that another solution might lead to the answer written in the form
\[
\sqrt{\frac{4 + \sqrt{15}}{6}}, \quad \sqrt{\frac{4 - \sqrt{15}}{6}}.
\]

15. There are $s - 1$ pairs of positive integers $a, b$ such that $a + b = s$. Therefore, the number of pairs of non-zero numbers such that $|x| + |y| \leq 100$ is equal to $4(1 + 2 + \cdots + 99) = 200 \cdot 99 = 19800$. Among pairs of integers such that $|x| + |y| \leq 100$ there are 200 such that $x = 0$ and $y \neq 0$, and 200 such that $x \neq 0$ and $y = 0$. The only remaining case is the pair $x = 0, y = 0$. In total we get $19800 + 400 + 1 = 20201$.

Another solution is just to use the answer in Problem 3.

16. Divide it by 2:
\[1/2 \sin \alpha - \sqrt{3}/2 \cos \alpha = \cos(\pi/3) \sin \alpha - \sin(\pi/3) \cos \alpha = \sin(\alpha - \pi/3).
\]
The minimum value of $\sin(\alpha - \pi/3)$ is $-1$, hence the minimal value of the original expression is $-2$.

17. $\log_5 n$ is rational if and only if $n$ is a power of 2. It follows that the sum in question is equal to $\frac{1}{5}(0 + 1 + 2 + \cdots + 10) = \frac{55}{2}$.

18. Note that $3 + 2\sqrt{2} = 1 + 2\sqrt{2} + 2 = (1 + \sqrt{2})^2$, and $3 - 2\sqrt{2} = (\sqrt{2} - 1)^2$. It follows that the number in question is equal to
\[1 + \sqrt{2} - \sqrt{2} + 1 = 2.\]

19. If equation of the line is $y = ax + b$, then the $x$-coordinates of the points are roots of the polynomial $2x^4 + 7x^3 + 3x - 5 - ax - b = 2x^4 + 7x^3 + (3 - a)x + (-5 - b)$. Sum of roots of this polynomial is $-7/2$. 

\[2\]
20. $\angle MOB = \angle OBC = \angle OBM$, since $MN$ is parallel to $BC$, and $BO$ bisects $\angle CBA$. It follows that \( \triangle BMO \) is isosceles, so that $OM = MB$. We prove in the same way that $ON = NC$. It follows that perimeter of \( \triangle AMN \) is equal to $AB + AC = 12 + 18 = 30$.

21. We can take $P(n) = n + 3$ and $Q(n) = -n - 2$, since

\[
(n + 3)f(n + 1) - (n + 2)f(n) = (n + 3)(f(n) + (n + 1)!)) - (n + 2)f(n) = f(n) + (n + 1)! + (n + 1)! + (n + 2)! = f(n + 2).
\]