1. \[
\frac{2^{12} - 2^{10}}{8^{(14/3)}} = \frac{2^{10}(2^2 - 1)}{2^{14}} = \frac{3}{2^4} = \frac{3}{16}.
\]

2. \[
4^x + 4^{x+1} = 160
\]
\[
4^x + 4 \cdot 4^x = 160
\]
\[
5 \cdot 4^x = 160
\]
\[
4^x = 32
\]
\[
2^{2x} = 2^5
\]
\[
2x = 5
\]
\[
x = 5/2.
\]

3. Let \( F \) be the size of the original flock. Then the shepherd has
\[
F - \frac{2}{3}F + \frac{4}{5} \left( \frac{2}{3}F \right) = \left( 1 - \frac{2}{3} + \frac{8}{15} \right) F = \frac{13}{15}F
\]
remaining.

4. The prime can have no more than 4 digits and these digits are 1, 2 or 3. Possibilities 1+1+1+1=4 but 1111 is not prime, 1+1+2=4 giving 112,121,211 and 211 is prime, 1+3=4 giving 13 and 31 both are prime.

5. \[
\begin{align*}
(1 \oplus 2) \oplus c &= 1 \oplus (2 \oplus c) \\
\left( \frac{1}{1} + \frac{1}{2} \right) \oplus c &= 1 \oplus \left( \frac{1}{2} + \frac{1}{c} \right) \\
\frac{3}{2} \oplus c &= 1 \oplus \frac{c + 2}{2c} \\
\frac{2}{3} + \frac{1}{c} &= 1 + \frac{2c}{c + 2} \\
\frac{1}{c} - \frac{2c}{c + 2} &= \frac{1}{3} \\
c + 2 - 2c^2 &= \frac{1}{3}(c(c + 2))
\end{align*}
\]
\[ 3c + 6 - 6c^2 = c^2 + 2c \\
7c^2 - c - 6 = 0 \\
(7c + 6)(c - 1) = 0 \]
\[ c = 1, -6/7 . \]

6. There are 36 possible rolls. If one of the dice is a five the game is won. This occurs \( 6 + 6 - 1 = 11 \) ways. If a five is not rolled, one wins with a throw of \((1,1), (1,3), (3,1), (3,3)\). So the game is won in \(11+4=15\) ways. The probability of winning is \(15/36 = 5/12\).

7.
\[
n = 2 + 3 + 3^2 + 2 \cdot 3^3 + 3^4 + 2 \cdot 3^5 + 2 \cdot 3^6 \\
= 5 + 3^2 + 6 \cdot 3^2 + 3^4 + 6 \cdot 3^4 + 2 \cdot 3^6 \\
= 5 + 7 \cdot 9 + 7 \cdot 9^2 + 2 \cdot 9^3 \\
= 2775 \\
\]

8. The median score is 78. The number of scores between 73 and 83 inclusive is 8. Since \(8/37 = .216\ldots\) then the percent is 22.

9. Let \(s\) be the length of a side of the larger triangle and \(t\) the smaller one. Then \(3s + 3t = 45\) or \(s + t = 15\) and \(s^2 = 16t^2\). Hence,
\[
(15 - t)^2 = 16t^2 \\
225 - 30t + t^2 = 16t^2 \\
15t^2 + 30t - 225 = 0 \\
t^2 + 2t - 15 = 0 \\
(t + 5)(t - 3) = 0 \\
\]
and \(t = 3\) with \(s = 15 - t = 12\). The area of the larger triangle is
\[
n \frac{1}{2} \frac{\sqrt{3}}{2} s^2 = \frac{\sqrt{3}}{4} (12)^2 = \sqrt{3}(3)(12) = 36\sqrt{3} . \]

10. The number of subsets having 6 as their largest element is the number of subsets of \(\{1, 2, 3, 4, 5, 6\}\) that contain 6, namely \(2^3 = 32\). Similarly the number of subsets having 4 as their largest element is the number of subsets of \(\{1, 2, 3, 4\}\) that contain 4, namely \(2^3 = 8\). So \(8+32=40\).
11. $20! = 20 \cdot 19 \cdots 15 \cdot 10 \cdots 5 \cdot 2 \cdot 1$ ends in four zeros. So $10^4 = 10,000$ divides both $20!$ and $400,000$.

$400,000 = 40(10,000) = (8)(5)(10,000)$

8 divides $\frac{20!}{10000}$, but 5 does not. So the greatest common divisor of $20!$ and $400,000$ is $8(10,000) = 80,000$.

12.

\[
\begin{align*}
\frac{1}{x} + \frac{1}{y} &= \frac{1}{\frac{1}{x} + \frac{1}{y}} = \frac{1}{\frac{xy}{y + x}} = \frac{1}{7} \\
7x + 7y &= xy \\
y &= \frac{7x}{x - 7} = \frac{7(x - 7) + 49}{x - 7} = 7 + \frac{49}{x - 7}.
\end{align*}
\]

Since $y$ is a positive integer then $\frac{49}{x-7}$ is a positive integer. Hence $x - 7 = 1, 7$ or 49 or $x = 8, 14$ or 56. If $x = 8$, then $y = 56$. If $x = 14$, then $y = 14$. If $x = 56$, then $y = 8$. So $(8, 56), (14, 14), (56, 8)$.

13.

\[
\begin{align*}
m &= \sqrt{x + 43}, \ n = \sqrt{x + 16} \\
m^2 &= x + 43, \ n^2 = x + 16 \\
m^2 - n^2 &= 27 \\
(m + n)(m - n) &= 27 = 3^3
\end{align*}
\]

Since $m, n$ are positive integers and $m + n > m - n$, then one possibility is $m + n = 9, m - n = 3$ which gives $m = 6, n = 3$. But then $6 = \sqrt{x + 43}$ or $36 = x + 43$ with $x = -7 < 0$, not true. The other possibility is $m + n = 27, m - n = 1$ which gives $m = 14, n = 13, x = 153$. So $mn = (14)(13) = 182$.

14. Let $s = |AB| = |DC|$ and $t = |AD| = |BC|$.

\[
\text{Area of WXYZ} = st - \frac{1}{2} \left( \frac{1}{4} \right) (\frac{1}{2}s) - \frac{1}{2} \left( \frac{1}{2} \right) (\frac{1}{2}t) - \frac{1}{2} \left( \frac{1}{2} \right) (\frac{1}{2}t) - \frac{1}{2} \left( \frac{1}{2} \right) (\frac{1}{3}s) - \frac{1}{2} \left( \frac{1}{2} \right) (\frac{1}{3}s)
\]

\[
= st - \left( 1 - \frac{1}{16} - \frac{1}{8} - \frac{1}{12} - \frac{1}{4} \right) = st - \left( 1 - \frac{1}{16} - \frac{2}{16} - \frac{1}{12} - \frac{3}{12} \right)
\]

3
\[ \begin{align*}
\text{st} &= \left(1 - \frac{3}{16} - \frac{4}{12}\right) \\
\text{st} &= \left(1 - \frac{9}{48}\right) \\
\text{st} &= \left(1 - \frac{25}{48}\right) \\
\text{st} &= \frac{23}{48} \\
\text{st} &= \frac{23}{48} \text{(Area of ABCD)} .
\end{align*} \]

15. 
\[
(a + 1) + (a + 2) + \cdots + (a + k) = 105 \quad (k \geq 1) \\
ka + (1 + 2 + \cdots + k) = 105 \\
ka + \frac{k(k+1)}{2} = 105 \\
2ka + k(k+1) = 210 \\
k(2a + k + 1) = 210 = (2)(3)(5)(7)
\]

so \( k \) must be a positive divisor of 210 and there are \( 2^4 = 16 \) of these. It remains to see that if \( k \) is a divisor of 210 then an integer \( a \) exists. Solving for \( 2a \) gives

\[ 2a = \frac{210}{k} - (k + 1) . \]

If \( k \) is odd then both \( 210/k \) and \( k+1 \) are even, so \( a \) exists. If \( k \) is even then \( 210/k \) and \( k+1 \) are both odd. Hence \( (210/k) - (k+1) \) is even and \( a \) exists.

16. Let \( d \) be the one way distance, \( t_A \) the time against the wind and \( t_W \) the time with the wind.

\[
\begin{align*}
d &= 20t_A \\
d &= 30t_W \\
20t_A &= 30t_W \\
t_A &= \frac{3}{2}t_W .
\end{align*}
\]

The average speed over the round trip is

\[
\frac{2d}{t_A + t_W} = \frac{2d}{(3/2)t_W + t_W} = \frac{2d}{(5/2)t_w} = \frac{4}{5} \frac{d}{t_W} = \frac{4}{5}(30) = 24 .
\]
17.

\[ |k - (-1)| = 2|k - 3| \]
\[ (k + 1)^2 = 4(k - 3)^2 \]
\[ k^2 + 2k + 1 = 4k^2 - 24k + 36 \]
\[ 3k^2 - 26k + 35 = 0 \]

Using the quadratic formula one obtains

\[ k = \frac{26 \pm \sqrt{(26)^2 - 4(35)(3)}}{6} \]
\[ = \frac{26 \pm \sqrt{169 - 105}}{6} \]
\[ = \frac{13 \pm \sqrt{64}}{3} \]
\[ = \frac{13 \pm 8}{3} \]
\[ = \frac{5}{3} \text{ or } 7 \]

18. The area of the region is \( A = (4)(5) + (1)(6) = 26 \). So \( \frac{1}{2}A = 13 \). The line \( y = mx \) intersects the line \( x = 4 \) at the point \( (4, 4m) \) dividing R into two subregions and we want \( m \) so that each subregion has area 13. The “upper” subregion is a trapezoid with heights of lengths 5 and \( 5 - 4m \) and base of length 4. The area of the trapezoid is \( \frac{1}{2}(5 + 5 - 4m)4 = 2(10 - 4m) = 20 - 8m \). We want \( 20 - 8m = 13 \), so \( m = 7/8 \).

19. \( 80 = qn + 4 \) for some quotient \( q \). (We know \( n \) is an integer larger than 4.)

\[ 154 = 80 + 74 = qn + 4 + (qn + 4 - 6) = 2qn + 2 \]

So \( 2q \) is the quotient when \( n \) is divided into 154 with a remainder of 2.

20. \( y = 4x + 2 \), \( y = x + 3 \), \( y = -2x + 5 \) are with \( y \)-intercepts 2,3 and 5 respectively and slopes 4,1 and -2 respectively. So collectively their graphs form triangle PQR.
By inspection the maximum $g(x)$ occurs at the point Q, the intersection of the lines $y = x + 3$ and $y = -2x + 5$, namely when
\[
\begin{align*}
x + 3 &= -2x + 5 \\
3x &= 2 \\
x &= \frac{2}{3}.
\end{align*}
\]
Then the three numbers are $(8/3)+2$, $(2/3)+3$, $-(4/3)+5$ or $14/3, 11/3$, $11/3$ and $g(2/3) = 11/3$.

21. Enlarge the quadrilateral to the rectangle as pictured.

The area is
\[
(12)(20) - \frac{1}{2}(5)(12) - \frac{1}{2}(1)(20) - \frac{1}{2}(10)(11) = 145.
\]