1. The product of 180 and the positive integer $N$ is a perfect cube. If $N$ is larger than 2011, what is the smallest possible value of $N$?

2. Find the coefficient of $x^{2012}$ in the expansion of $(1 + x + x^2 + \cdots + x^{2011})(1 + x + x^2 + \cdots + x^{1006})^2$.

3. A permutation $a_1, a_2, a_3, a_4, a_5$ of the digits 1, 2, 3, 4, 5 is heavy-tailed if $a_1 + a_2 < a_4 + a_5$. Find the number of heavy-tailed permutations.

4. Points $A$ and $B$ are on a circle of radius 5 and $AB = 6$. Point $C$ is the midpoint of the minor arc $AB$. Find the length of line segment $AC$.

5. Let $T$ be a linear map on the set of polynomials, that is, for any polynomials $p_1$ and $p_2$ and any constant $c$, $T(p_1 + p_2) = T(p_1) + T(p_2)$ and $T(cp_1) = cT(p_1)$. If $T(1 + x) = x + 2$, $T(x^2 + x) = x$, and $T(1 - 2x^2) = 3$, what is $T(x^2)$?

6. Point $A$ is chosen at random from the line segment joining $(0, 0)$ and $(2, 0)$ as the center of a circle of radius 1. Point $B$ is chosen at random from the line segment joining $(0, 1)$ and $(2, 1)$ as the center of another circle of radius 1. What is the probability that the two circles intersect?

7. Compute $\sin(105^\circ) \cos(15^\circ)$. Express your answer in simplest terms.

8. Compute $\lim_{x \to 0} \frac{\cos x \sin x - \tan x}{x^2 \sin x}$.

9. The circle shown below is tangent to the parabola $y = x^2$ at the point $(\sqrt{2}, 2)$ and the $x$-axis. Find the radius of the circle.

10. Let $a \in N$ and $b \in N$ be natural numbers, $A = \{x \in R \mid 5x - a \leq 0\}$, and $B = \{x \in R \mid 6x - b > 0\}$. Find the number of pairs $(a, b)$ such that $A \cap B \cap N = \{2, 3, 4\}$. 


11. Let \( h(x) = xg(x) \), where \( g = f^{-1} \). Use the table of values below to find \( h'(5) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

12. \( ABCD \) is a square of side length 1m. At the four vertices, four circular arcs are drawn, each with radius 1m, as shown in the figure below. Find the area of the shaded region.

![Diagram of square with circular arcs]

13. Line \( \ell \) is tangent to the curve \( x^{2/3} + y^{2/3} = 9 \), \( x, y > 0 \). If \( A \) is the \( x \)-intercept of \( \ell \) and \( B \) is the \( y \)-intercept of \( \ell \), find the length of the line segment \( AB \).

14. Find the sum of all values of \( c \) such that the function
\[
 f(x) = \begin{cases} 
 x^2 + c^2x - 1 & \text{if } x \leq 1 \\
 cx^2 + 5x + 1 & \text{if } x > 1 
\end{cases}
\]
is differentiable at \( x = 1 \).

15. Let \( D \) be the region bounded by the \( x \)-axis, the graph of \( y = \sqrt{x} \), and a line tangent to the curve \( y = \sqrt{x} \). If the area of \( D \) is \( \frac{2}{3} \), find the \( x \)-coordinate of the point where the line is tangent to the graph.

16. Let \( f(x) = \begin{cases} 
 \sqrt{4x - x^2} & \text{if } 0 \leq x \leq 4 \\
 4 - x & \text{if } x > 4 
\end{cases} \).

Compute \( \int_{0}^{6} f(x) \, dx \).

17. Define a selfish set to be a set which has its own cardinality (number of elements) as an element. Define a minimal selfish set to be a selfish set none of whose proper subsets are selfish. For example, the sets \( \{1, 2\} \) and \( \{2, 5\} \) are selfish sets, but only \( \{2, 5\} \) is a minimal selfish set. Given \( A = \{1, 2, 3, \ldots, 11\} \), how many subsets of \( A \) are minimal selfish sets?

18. Lines \( K, L, \) and \( T \) are shown below. If \( L \) is the line \( x - 3y = -2 \) and \( T \) is the line \( 8x - 4y = 9 \), find the equation for \( K \) in the form \( Ax + By = C \) where \( A, B, \) and \( C \) are relatively prime integers and \( A > 0 \).