Directions: If units are involved, include them in your answer.

1. Mary drives $x$ km at an average speed of 60 km/h. She returns by a different route, which is 5 km shorter, at an average speed of 50 km/h. The total time for both journeys is 1 hour 33 minutes. Find the distance $x$.

2. If $1 \leq x \leq 3$ and $2 \leq y \leq 7$, what is the smallest value of $\frac{y}{x} - \frac{x}{y}$ in simplest terms?

3. An isosceles right triangle is removed from each corner of a square piece of paper so that a rectangle with a diagonal of length $5\sqrt{6}$ ft remains. What is the total area of the removed pieces?

4. The sum of two natural numbers $a$ and $b$ is equal to 153. What is the largest possible value of their greatest common divisor, gcd$(a, b)$?

5. Increasing $x$ by $y\%$ gives 30, increasing $y$ by $x\%$ gives 25. Find $x$.

6. The radius of the big circle is 2 cm. Each of four smaller circles has radius 1 cm. Find the total area of the shaded parts.

7. Determine all positive integers $n > 3$ for which $n^3 - 3$ is divisible by $n - 3$.

8. Find all three digit numbers such that if you increase a number by 3, the sum of its digits decreases three times.

9. Consider a trapezoid $ABCD$ with the lengths of bases $AB$ and $CD$ being equal to $a$ and $b$ ($a > b$), respectively. A point $E$ on side $AD$ and a point $F$ on side $BC$ are such that $AE : ED = BF : FC = m : n$. Determine the distance between $E$ and $F$.

10. In the figure below sector $AOB$ is a quadrant of a circle of radius $R$. The arcs $\overline{AC}, \overline{CD}, \overline{DB}$ are equal in length, and $CE \parallel DF \parallel BO$. What fraction of the circle’s area is shaded?
11. Solve the system of equations

\[
\begin{align*}
xy + x + y &= 19 \\
x^2y + xy^2 &= 84
\end{align*}
\]

12. \(ABCD\) is a square of side 1 m. With its vertices as centers, four circular arcs, each of radius 1 m, are drawn as shown. Find the shaded area.

13. Solve the equation \(\sqrt{1 + mx} = x + \sqrt{1 - mx}\) for \(x\) and determine all possible values of a real parameter \(m\) for which a real solution exists.

14. The sequence 1, 8, 22, 43, ... has the property that the differences of two neighboring terms \(a_n - a_{n-1}\) (\(n = 2, 3, \ldots\)) form the arithmetic sequence: 7, 14, 21, ... Find the number of the term 35351 of the original sequence.

15. Given triangle \(ABC\) with area \(S\), the median \(BD\) is drawn. A point \(E\) is on \(BD\) and such that \(DE = \left(\frac{1}{4}\right)BD\). The line \(AE\) is drawn, intersecting \(BC\) at a point \(F\). Find the area of the triangle \(AFD\).

16. Let \(f(x) = \frac{cx}{x^2+3}\) where \(x \neq -3/2\). Find all values of \(c\), if any, for which \(f(f(x)) = x\) for all \(x \neq -3/2\).

17. A cube of volume 64 cubic centimeters is inscribed in a sphere. What is the surface area of this sphere?

18. Six consecutive integers were written on a blackboard. When one of them is erased, the sum of the remaining five is 2011. What number was erased?

19. Solve the equation \(x + a^3 = \sqrt[3]{a - x}\) (where \(a > 0\) is a parameter) for \(x\) in terms of \(a\).

20. The number \(\sqrt[3]{45 + 29\sqrt{2}} + \sqrt[3]{45 - 29\sqrt{2}}\) is a rational number (though it does not immediately look like one). Confirm this fact by rewriting the number as a fraction \(\frac{p}{q}\) of two integers \(p, q\) in lowest terms.